

## A Problem Unique to Long-Barreled Rifles

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### 1. Introduction

An interesting phenomenon occurs when firing the extremely long-barreled target rifles often used for long-range and extreme-long-range (ELR) shooting. If the rifle barrel is too long and is “free floated,” its fired bullets will always exit the muzzle of the barrel with a positive pitch (nose up) attitude of their spin-axes and with a positive (upward) pitch-rate due to barrel vibration timing constraints. The reverse aerodynamics of the bullet while transiting the muzzle-blast zone can only amplify these non-zero nose-up attitudes and rates. For best rifle accuracy and bullet flight performance, we usually strive to launch our bullets with zero initial yaw and yaw-rate. We can tune our loads to do just that with shorter rifle barrels.

Longer rifle barrels vibrate at lower natural frequencies than shorter barrels. If the barrel’s critical **Mode 2** transverse vibration frequency is too low, its initial halt and reversal from downward to upward motion at the muzzle can only happen after bullet release, so the muzzle is still sweeping downward at that critical instant. That downward muzzle motion causes the released bullet to be tipped downward at its rear—and upward at its nose.

The effect of this inevitable nose-up initial attitude and upward initial pitch-rate in early ballistic flight is to *facilitate* firing whenever the crosswind at the firing point is blowing from right-to-left whenever we are firing with a barrel made with the more commonly used right-hand twist direction; and **vice-versa**. The results will not be so favorable when firing this same long-barreled rifle through left-to-right crosswinds.

Think of the ballistics wind axes plot of the bullet’s spin-axis direction plotted in aircraft-type pitch and yaw coordinates. The bullet commences ballistic flight with its spin-axis pointed along the +V direction of flight (the origin of the plot) if it starts ideally with zero initial yaw and yaw-rate. Within the first half coning cycle, the spin-axis is circling the eye of the apparent wind—wherever it lies. The radius of that circle is the first maximum yaw angle.

The right-hand spinning bullet can only cone clockwise about the eye of the apparent wind as seen from behind. If that eye of the wind is horizontally rightward of the +V muzzle direction, commencing that coning motion is facilitated by a slight upward initial yaw and yaw-rate measured from the origin. But if that eye of the wind is leftward of the muzzle, the spin-axis must first spiral out rightward before it can circle back to the left, making for a significantly larger first maximum coning angle-of-attack.

For a favored combination, the total air-drag on the bullet is *decreased* due to coning with a smaller initial coning angle-of-attack; crosswind drift (windage correction) is decreased due to this reduced air-drag; the effective Ballistic Coefficient (BC) of the bullet is maximized; and both an initial aerodynamic jump and the long-range spin-drift partially compensate the (already reduced) windage aiming correction needed. The rifleman and coach have a good day at the range.

However, an unfavored combination causes a much larger initial coning angle which results in much greater yaw-drag (which increases with the *square* of that coning angle in combining into the total drag); significantly greater crosswind sensitivity (due to that increased air-drag); increased windage correction (due to that increased air-drag and additive horizontal aerodynamic jump and spin-drift corrections); and increased flight times to the target (due to a significantly lower effective BC) of the fired bullets. Any left-to-right crosswind at the firing point favors those riflemen using left-hand twist longer rifle barrels. Those firing right-hand twist barrels will usually not do as well.

Conventional length rifle barrels allow the load to be carefully tuned with respect to the vertical barrel muzzle motions at bullet exit. Competitive short-range benchrest riflemen arrive early at the range so they can tune their load to prevailing range conditions for the upcoming match. Crosswind direction along the firing line is one of those conditions, whether or not the shooter consciously realizes it.

The initial pitch attitudes of bullets fired from shorter-barreled rifles can be either upward or downward (or, ideally, tuned to zero). The initial pitch-rate is always in the direction of increasing the magnitude of any non-zero initial pitch attitude.

The breakpoint in barrel lengths seems to be in the **32-inch** range for our 50 BMG example cartridge discussed here, but that maximum length depends upon many factors. For example, one could select a heavier bullet or a lighter load to increase barrel dwell time into the tunable range for their slightly longer rifle barrel. Or, one could select a much heavier barrel profile, and put wheels on it, to increase its natural vibration frequencies. We will show that barrel vibration rates are much more sensitive to barrel length than to barrel stiffness or to its mass per unit length.

Aside from using heavier bullets or lighter loads with their longer barrel dwell times, or selecting a shorter barrel to increase its critical **Mode 2** vibration frequency, the use of a barrel mounted bipod support attached to the long barrel near its muzzle is the only other rifle design approach which comes to mind for ameliorating this firing problem when using extra-long rifle barrels. Those riflemen most likely to experience this long-barrel problem are ELR shooters, especially 50-caliber shooters, striving for maximum muzzle velocities and across-the-course target riflemen also seeking an increased sight radius and to improve the “hang” of their target rifle in off-hand shooting.

## 2. Some Relevant Physical Principles

The physical principle of Conservation of Momentum requires that both the linear momentum and angular momentum of the fired rifle bullet be *continuous variables* at all times throughout its flight. This means that no step-change discontinuities can occur in these vector momentum values between the successive flight regimes of the fired bullet. The linear momentum  $\mathbf{p}$  of the constant-mass bullet is  $\mathbf{p} = m \cdot d\mathbf{X}/dt = m \cdot \mathbf{V}$  where  $\mathbf{X}$  represents the vector position of the CG of the bullet in a (pseudo-inertial) earth-fixed coordinate system. [We separately calculate a “Coriolis” pseudo-force to compensate for the earth’s sidereal rotation in propagating trajectories.] This linear momentum vector quantity  $\mathbf{p}$  is carried over unchanged in either its direction or magnitude across the boundary between the muzzle-blast region and ballistic flight. However, the second and higher derivatives of  $\mathbf{X}$ -position versus time may be *discontinuous* as forces acting upon the bullet are applied or changed step-wise over flight time.

If the rifle bullet is simply spinning about its principal (longitudinal)  $\mathbf{x}$ -axis, its angular momentum  $\mathbf{L}$  is given by  $\mathbf{L} = \mathbf{I}_x \cdot \boldsymbol{\omega}$ , where  $\boldsymbol{\omega}$  is the instantaneous spin-rate of that bullet in radians per second and  $\mathbf{I}_x$  is the second moment of inertia of the mass distribution of the bullet about its longitudinal (principal) axis. The second moment of inertia has units of mass times distance squared. This  $\mathbf{x}$ -axis of symmetry of the (ideal) bullet is a principal axis of inertia because it has the *minimum* possible second moment for its mass distribution. The vector angular momentum quantity  $\mathbf{L}$  has the direction of  $\boldsymbol{\omega}$  and must be *continuous* across all flight segment boundaries.

More generally however, the total vector angular momentum  $\mathbf{L} = \mathbf{L}_x + \mathbf{L}_y$  of the rifle bullet which must be *conserved* has to allow for the incorporation of a small, but non-zero transverse angular momentum component,  $\mathbf{L}_y = \mathbf{I}_y \cdot (\boldsymbol{\alpha} \cdot \text{dot})$  perpendicular to  $\mathbf{L}_x = \mathbf{I}_x \cdot \boldsymbol{\omega}$ , where  $\boldsymbol{\alpha} \cdot \text{dot} = d\boldsymbol{\alpha}/dt$  is a much smaller vector angular rate of “tumbling” of the bullet in radians per second and  $\mathbf{I}_y$  is the (much larger) second moment of the bullet’s mass distribution about any transverse  $\mathbf{y}$ -axis through the CG of the bullet. This initial yaw-rate vector  $\boldsymbol{\alpha} \cdot \text{dot}$  can be thought of as a two-dimensional vector velocity of the bullet’s spin-axis pointing direction as shown in wind-axes plots of its aircraft-type pitching and yawing motions at the beginning of ballistic flight. Thus, the yaw-rate  $\boldsymbol{\alpha} \cdot \text{dot}$  must also be carried over unchanged from the muzzle-blast zone into ballistic flight so that the bullet’s total angular momentum is conserved.

Here we are often using the term “yaw” to mean “aeroballistic yaw” for axisymmetric spin-stabilized projectiles. For small yaw angles, it is the root-sum-squares (RSS) of the standard “aircraft type” pitch and yaw attitudes. The “roll” orientation of this aeroballistic yaw attitude is not normally specified, as it varies continually with the coning motion of the flying bullet.

### 3. Muzzle-Blast Flight Regime

The final linear momentum  $\mathbf{p}$  and angular momentum  $\mathbf{L}$  of the bullet after passing through the muzzle-blast zone are determined by (1) their initial values just after the bullet clears the muzzle, (2) the flow characteristics of the hot gasses and particulates exiting the muzzle behind the bullet, and (3) the reverse aerodynamics of the bullet itself. The muzzle-blast flight regime commences with *bullet release* from the muzzle and ends after

about **15 to 20 calibers** of flight distance when the supersonic bullet has completely penetrated the leading Mach 1.0 shockwave of the muzzle-blast. For a heavy 50-caliber rifle bullet fired from a large volume, but relatively low muzzle pressure, 50-Caliber Browning Machine Gun (50 BMG) cartridge, the muzzle-blast zone extends about **7.5 inches (15 calibers)** ahead of the muzzle. If our 50-caliber bullet averages **2730 fps** through this muzzle-blast zone, its total transit time would be just **229 microseconds**.

### 3.1 Mechanics of Bullet Release from the Muzzle

*Bullet release* occurs with the ending of any physical (mechanical) contact between any bullet surfaces and any part of the rifle barrel. The high-pressure gas sealing of the bore by the bullet is also broken at this same time. For our example 850-grain boattailed 50-caliber copper bullet, about 85-percent of its conical boattail is still behind the crown of the muzzle at the instant of bullet release.

Here we are using 235 grains of Hodgdon's H-50BMG propellant to fire this example 850-grain copper bullet at a muzzle velocity of 2722 fps from a 44-inch barrel chambered in 50 BMG. Bullet release occurs at about **2.2 milliseconds** after primer ignition. The movement of the muzzle end of the rifle barrel at the instant of bullet release is determined both by the rifle's reactions to the recoil impulse caused by accelerating the bullet and powder down the bore and by vertical standing wave transverse barrel vibrations.

We will discuss the effects of muzzle motion on the released bullet in detail for this 50 BMG rifle as we work through this example.

### 3.2 Recoil Effects at the Muzzle

If the vector sums of the linear momenta and angular momenta of all parts of the rifle, bullet, and powder system are conveniently taken to be zero just before firing, then they must always vector-sum to zero over that closed system, including at the instant of bullet release.

An 850-grain 50-caliber bullet fired with 49,000 psi peak base pressure is accelerated by a peak force (pressure times area) of **10,033 pounds**. This same size “equal and opposite” reaction force acts on the rifle through the center of its breech face. If the CG of the rifle and all of its attachments is **1 inch** below the axis of the bore, a momentary (peak) torque of **836 foot-pounds** is bending upward on the attached rear end of the barrel. The receiver end of the rifle barrel is strongly torqued upward about its receiver attachment, assuming that the rifle is a target rifle with its barrel properly “free floated” clear of the fore-end of the stock. For a conventional shoulder-fired rifle, its horizontal and vertical support points and the CG of the rifle assembly itself all lie directly below the line-of-action of the recoil momentum vector, acting parallel to the axis of the bore and through the center of the breech face, and producing this upward recoil torque on the barrel attachment point. This momentary recoil torque vector points horizontally rightward from the barrel-to-receiver attachment by the “right hand rule” convention in handling vectors.

As the bore of the rifle barrel in flat firing is internally pressurized behind the moving bullet, it generates a small *additional* upward torque about its receiver attachment due to “pressure stiffening” of the hollow barrel against the gravity droop it had before firing. This effect is probably greatest for lightweight magnum hunting rifles having long, slender barrels. For a well made target rifle, all of these forces and moments should be restricted to acting only in a vertical plane through the axis of the bore.

The upward barrel bending impulse has exactly the timing of the rifle’s recoil force impulse, which in turn is determined by the timing of the base pressure driving the bullet; starting with initial bullet acceleration right after bullet engraving by the rifling lands, building rapidly to a peak when the bullet experiences peak base pressure after moving only 2.75 inches forward, and then decreasing more slowly and continuing weakly even well after bullet release.

The rearmost portion of the barrel undergoing this torque impulse is bent upward. The bore axis, however, is curving downward due to inertial resistance of the barrel mass located forward of any particular point along the barrel and due to shear-wave propagation delays (at 3250 meters per second in our steel barrel). Consequently, this recoil-forced downward

curvature decreases continuously from breech end to muzzle end. The overall effect is for the barrel to assume a shape which closely matches a Mode 2 free vibration standing wave shape, especially in the front (muzzle) half of the barrel. A point 80-percent of the way to the muzzle remains on the original axis of the receiver as it was before firing. In the vicinity of that 80-percent point the barrel remains straight (with little to no recoil induced curvature), but the whole front half of the barrel points slightly downward relative to the original receiver axis direction.

This means that, as long as the barrel continues actively being bent vertically upward at its receiver end, at least the front 20-percent of the barrel and the muzzle are sweeping vertically *downward*. Any and all higher frequency natural barrel vibration modes are simply superimposed upon this recoil-forced low-frequency (Mode 2) barrel shape and its downward motion of the muzzle. Calculations of base pressures driving this example bullet in the bore indicate that the barrel bending recoil force at bullet release is only about 17-percent of its peak value occurring about 1.52 milliseconds earlier. The recoil forcing of this “almost Mode 2 shaped” barrel bend does not even cease completely with bullet exit from the muzzle, but weakly continues on due to the rocket nozzle effect of the propellant gasses exiting behind the bullet until those gasses are exhausted well after the bullet has cleared the muzzle-blast flight region.

4. Natural Transverse Vibration Mode Frequencies and Shapes  
Beginning effectively when the base pressure nears its peak at **680  $\mu\text{sec}$  (microseconds)** into the firing process, this vertically upward bending torque impulse initiates a peak shear distortion at the breech end of the steel barrel which propagates as a transverse ***shear-wave*** toward the muzzle at **3250 meters per second** (128,000 inches per second). Starting at the end location of torque application at the time of peak base pressure, this shear wave reaches the muzzle of our 44-inch rifle barrel **344  $\mu\text{sec}$**  later, reflects, and makes **6.3802** total one-way trips back and forth along the 44-inch barrel by the time of muzzle exit of the fired bullet (at  **$t = 2194 \mu\text{sec}$** ). The shear wave reflections upon encountering the impedance changes at each end of the rifle barrel may involve a phase reversal (at the breech-end node) and the dissipation of some portion of the wave energy

into longitudinal acoustic pressure waves and into the excitation of the various higher-frequency natural vibration modes.

Note that Mode 1 vibrations with the whole barrel moving as a unit at only **23.8 hertz** do not occur in free-floated rifle barrels because (1) the initial barrel distortion does not assume this shape and (2) the excitation spectrum does not contain significant energy at that low frequency. The impulsive recoil torque acting on the barrel ends too quickly to excite this mode, as anyone who has handled a flexible flyrod would understand.

These reciprocating transverse waves reinforce into standing vibrational waves of several different natural modes and frequencies, with the lowest frequency mode actually occurring (Mode 2, with a natural resonant frequency of **151.4 hertz**) having just one extra node, in addition to the point of attachment which is always a vibration node. This additional node for transverse Mode 2 is located at **78.34**-percent of the distance from breech to muzzle of a uniform barrel of length **L**.

As noted earlier, the recoil induced distortion of the front half of the rifle barrel closely matches the shape of a Mode 2 standing wave amplitude pattern starting vertically upward from its breech-end node. The difference is that the breech-end torque is being actively driven by recoil versus being passively clamped during free vibrations (i.e., different breech-end boundary conditions). It is the recovery of the muzzle end toward its neutral point at its natural Mode 2 oscillation rate (here **151.4 hz**) which first slows and then reverses the recoil driven downward initial motion of that muzzle end.

The frontmost nodes for each higher-frequency transverse mode are successively nearer to the muzzle. Mode 6 with a frequency of **2018 hertz** is the highest mode likely to be excited for this very long (44 inches), rather slender (1.038-inch average diameter), 8.0-pound rifle barrel. The peak frequency in the recoil excitation spectrum is at about  $1/(4 \times 680 \mu\text{sec}) = 367 \text{ hertz}$ , with asymmetric Gaussian-shaped roll-off on either side. This peak excitation frequency falls between the frequencies of natural Mode 2 (**151.4 hz**) and Mode 3 (**416.9 hz**).

Unless a particular natural mode vibration is being “pumped” by a continuing driving force having a resonant frequency, the muzzle



amplitudes of each higher-frequency mode are normally smaller than those of the lower-frequency modes. The excitation is impulsive in rifle firing.

Shorter, stiffer barrels have higher mode frequencies  $f_n$  for all of their natural transverse vibration modes. We model the rifle barrel here as having uniform cross-section throughout its length because its mode shapes and frequencies are then given in handbooks for mechanical engineers, but these values are closely matched for moderately tapered barrel contours (such as the Heavy Varmint profile) when an average OD is used. We calculate this average OD from the weight (or mass) of the rifle barrel and its material density.

Consider a uniform, long, slender, cylindrical steel rifle barrel of length  $L$ , diameter  $D$ , and caliber  $d$  rigidly clamped at one end and free at its muzzle end.

[Insert Figure 1. Spreadsheet Calculation of Barrel Mode Frequencies.]

This exercise shows a calculation of the natural (or “free”) vibration mode frequencies  $f_n$  of a 50-caliber ( $d = 0.510$  inch, **or 0.01295 m**) target rifle barrel which is 44-inches (**1.118 m**) in length  $L$ , having an average diameter  $D$  of 1.038-inch (**0.0264 m**). The example barrel weighs **8.0 pounds**.

The mode frequencies  $f_n$  are most conveniently calculated using metric system MKS units (meters, kilograms, and seconds). An active Excel spreadsheet is freely available from the author allowing rapid calculation of these natural vibration modes for any set of barrel dimensions with that barrel’s material properties (which are assumed to be isotropic) being specified. We can more simply calculate these mode shapes and frequencies for a barrel of uniform diameter and having no muzzle attachment. These results will be close approximations for moderately tapered barrels such as Heavy Varmint or Palma profiles. We will not consider the effects of muzzle attachments here.

Our example AISI 4140 Chrome-Moly steel 50-caliber barrel material has a density  $\rho$  of **7850 kilograms per cubic meter** (490 pounds per cubic foot)

and Young's Modulus of Elasticity **E** of **205 Giga-Pascals** (29.7 million psi). Poisson's Ratio is **0.29** for this steel material.

The **cross-sectional area A** of the barrel is

$$A = (\pi/4)*(D^2 - d^2) = 4.1367 \times 10^{-4} \text{ meters}^2$$

The **mass per unit length** of the barrel is **A\*ρ = 3.2469 kg/meter**, and its **flexural rigidity** is **E\*I = 9130.73 kg meters<sup>3</sup> per seconds<sup>2</sup>**, where **I** is the cross-sectional **second moment of area** of the barrel considered as a vertically loaded cantilever beam:

$$I = (\pi/32)*(D^4 - d^4) = 4.4589 \times 10^{-8} \text{ meters}^4$$

Then, with barrel length **L** specified in meters, the resonant frequencies **f<sub>n</sub>** in hertz for transverse vibration mode **n** (**n = 1, 2, 3, 4, 5, 6....**) are given from:

$$f_n = [1/(2*\pi)]*(a_n/L^2)*SQRT[E*I/(A*\rho)]$$

where	<b>a<sub>1</sub> = 3.52</b>	<b>f<sub>1</sub> = 23.8 hertz</b>
	<b>a<sub>2</sub> = 22.4</b>	<b>f<sub>2</sub> = 151.4 hertz</b>
	<b>a<sub>3</sub> = 61.7</b>	<b>f<sub>3</sub> = 416.9 hertz</b>
	<b>a<sub>4</sub> = 121.0</b>	<b>f<sub>4</sub> = 817.6 hertz</b>
	<b>a<sub>5</sub> = 199.9</b>	<b>f<sub>5</sub> = 1350.8 hertz</b>
	<b>a<sub>6</sub> = 298.6</b>	<b>f<sub>6</sub> = 2017.7 hertz</b>

for **n > 5**, **a<sub>n</sub> = [(2\*n - 1)\*π/2]<sup>2</sup>**

These mode values of **a<sub>n</sub>** for clamped-free end conditions are taken from the handbook *Formulas for Natural Frequency and Mode Shape* by R. D. Blevins (Van Nostrand, 1979).

Note from this formulation for the natural mode frequencies **f<sub>n</sub>** that, while these natural mode frequencies all *increase* directly with barrel diameter **D**, they *decrease* much more strongly with the *square* of barrel length **L**. That is, increasing barrel length **L** by, say, **10-percent** would require a **21-percent** increase in diameter **D** to offset that change, and that new **10-percent** longer barrel would have to weigh **46.4 percent** more. Barrel length **L** is the primary characteristic determining its natural frequencies **f<sub>n</sub>**.

This same handbook gives the **shape** of the **amplitude envelope** for transverse **Mode 2** standing wave vibrations at  $f_2 = 151.4$  hertz of a cantilever (clamped-free ends) beam of length  $L = 44$  inches as:

$$\lambda_2 = 4.69409113$$

$$\sigma_2 = 1.018467319$$

$$\Theta(X) = \lambda_2 * X/L \quad (\text{for } 0.0 \leq X \leq 1.0)$$

$$Y_2(X) = (A_2/2) * \{ \cosh \Theta(X) - \cos \Theta(X) - \sigma_2 * [\sinh \Theta(X) - \sin \Theta(X)] \}$$

where  $Y_2(X)$  gives the relative **Mode 2** vibration amplitudes along  $L$ , and

$A_2$  = Amplitude of sinusoidal  $f_2$  vibration at the free (**muzzle**) end.

Hyperbolic trigonometric functions frequently arise in the study of mechanical vibrations with their various boundary conditions. Similar values and equations are given in the handbooks for the other mode shapes or with different boundary conditions (clamped, free, pinned, or sliding).

[Insert Figure 2. Graph of Mode 2 Shape for Cantilever Beam.]

The amplitude  $A_2$  occurring at the muzzle is the largest **Mode 2** vibration amplitude occurring anywhere along the rifle barrel. Each local particle of the barrel material is vibrating sinusoidally in a vertical plane at frequency  $f_2$  and with amplitude  $Y_2(X)$ .

This Mode 2 transverse vibration shape defines **almost exactly** the shape of the recoil induced distortion for the **front (muzzle) half** of a rifle barrel of length  $L$  and uniform section. The breech halves of the two shapes are just vaguely similar. The “zero crossing” of each shape occurs at  $X = 0.7834 * L$  which is the location of the second node for **Mode 2** barrel vibrations and at the point  $X = 0.80 * L$  on the rifle barrel which remains fixed on the receiver axis under recoil torque. This trivial difference in node locations explains why natural **Mode 2** vibrations are critically important for rifle barrels.

#### 4.1 The Combined Muzzle Motions at Bullet Release

The free, muzzle end of the plain rifle barrel vibrates vertically as an antinode for all the natural transverse vibration modes. It moves as the algebraic sum of all of those sinusoidal mode frequency motions, weighted by their respective mode amplitudes. We can best determine the dominant mode at the muzzle by test firing to find the more accurate shot groupings at the rifle barrel's successive vertical motion reversal points.

For a given conventional length rifle barrel, its dominant high-frequency transverse vibration mode at the muzzle is likely to be Mode 4, Mode 5, or Mode 6 and to be resonant at up to 4 to 6(+) kilohertz for a short, stiff barrel such as a 100-yard benchrest competition rifle barrel. For best accuracy in the precision rifle sports, the ammunition must be “tuned” to match the rifle barrel during load development by varying the barrel dwell time of the fired bullets and shooting an Audette ladder with incrementing powder charges at short to medium range (to minimize the effects of varying gravity drop with varying muzzle velocities).

Keep in mind, however, that even when the muzzle is “stationary” while reversing its dominant high-frequency oscillation, the muzzle may still be sweeping downward inertially (relative to the fixed earth) if its (**Mode 2** rate) recovery from its initial recoil driven shape distortion is sufficiently slow.

#### 4.2 Timing of Muzzle Motions

Here we are using the same time scale as does the interior ballistics program, QuickLOAD®; i.e., start time **t = 0** is when chamber pressure, which rises sharply, reaches **10-percent** of its eventual peak chamber pressure.

According to QuickLOAD (QL) calculations for this example 50 BMG rifle cartridge, the peak base pressure occurs at **680 µsec**. For our purposes in determining muzzle event timing, we can consider the upward torque impulse on the rear of the barrel to occur *instantaneously* at **t = 680 µsec**. when the rifling engraved bullet has moved **2.75 inches** into the bore.

The muzzle end of the 44-inch steel barrel begins to experience recoil torque about **44/128,000 ips = 344 µsec later**, at **t = 1024 µsec**, due to the “signaling delay” through **44-inches** of barrel steel. The shear-wave

propagates along this slender steel rod at **3250 m/sec** which is **128,000 ips**.

At this time, **t = 1024 μsec**, the muzzle **20-percent** portion of the barrel *begins* “whipping” downward inertially, strongly driven by the large recoil torque impulse, *and* a similarly shaped **Mode 2** transverse vibration is initially excited. This initial **Mode 2** vibratory motion of the muzzle starts at **t = 1024 μsec** as a downward angular sweeping motion of the muzzle involving the front **50-percent** of a uniform-section barrel. The motion of the muzzle itself is *sinusoidal*, *starting downward from its neutral position* at **t = 1024 μsec**, and then subsequently continuing to oscillate at its **Mode 2** frequency of **151.4 hertz**. The recoil impulse initiates a downward “whipping” motion at the muzzle at **t = 1024 μsec**, and the **Mode 2** elastic response limits that downward motion and controls the timing of its rebound.

After the first *quarter wave* (90 degrees) of this **Mode 2 (151.4 hertz)** elastic vibratory motion, which requires **1652 μsec**, the muzzle slows to a momentary stop in its vertical motion relative to the fixed earth. This first *halting* of muzzle motion occurs at time **t<sub>H</sub>** given in microseconds by:

$$t_H = t(\text{Peak Base Press. in } \mu\text{sec}) + L/(0.003250 \text{ m}/\mu\text{sec}) + 10^6/(4*f_2 \text{ hz})$$

$$\text{or } t_H = 680 + 344 + 1652 = 2676 \mu\text{sec}.$$

After this momentary halt, the muzzle begins rebounding *upward* in a sinusoidal pattern for the next half (180 degrees) of the first Mode 2 cycle (here **3303 μsec**), continuing long after the bullet has departed the muzzle-blast zone. This second halt in **151.4 hz** muzzle motion occurs just as high above its neutral position as the first halt was below it.

According to QL, our 850-grain rifle bullet exited the muzzle of our 44-inch barrel at **t = 2194 μsec**, well within the time interval (between **1024 μsec** and **2676 μsec**) during which the front **20-percent** of the barrel is sweeping its pointing direction *downward*. In fact, in this example, bullet release occurs at **70.7-percent** of this time interval, when the vertical downward velocity of the muzzle has slowed to **44.4-percent** of its maximum vibration movement rate (estimated at about **6 ips**). The forward speed of the bullet is calculated in QL to be **2722 fps** at release from the muzzle.

If QL indicates that your bullet's dwell time in the barrel is significantly less than  $t_h$ , the muzzle's **first halt time**, then one *cannot* tune out this downward vertical muzzle motion using minor variations in loading data. The amplitude  $A_2$  of the recoil distortion and the **Mode 2** vibration response is much larger than any combination of higher-frequency vibration amplitudes ( $A_3$ ,  $A_4$ ,  $A_5$ ,  $A_6$ , etc.) could match or overpower.

[Insert Figure 3. Table of Barrel Dwell versus Muzzle Reversal Times.]

### 4.3 Effects of Muzzle Motions upon Released Bullet

So, what does this downward motion of the muzzle imply about the bullet release process? If the engraved length of our example 338-caliber rifle bullet is **1.050 inch**, its CG is right at the base of the ogive (at the foremost end of the engraving), and it exits at a forward speed of **2722 fps**, the muzzle release time  $t_R$  is

$$t_R = (1.050)/(12 \cdot 2722) = 32.2 \text{ } \mu\text{sec}$$

from CG clearance to complete bullet release from the muzzle.

The maximum vertical muzzle speed  $V_m$  can be estimated based on other studies (*Rifle Accuracy Facts* by Harold Vaughn, Precision Shooting, 1998) to be up to about **6 ips** for this long **44-inch** barrel, which corresponds to a vibration amplitude of  $6 \text{ ips} / (2 \cdot \pi \cdot 151.4 \text{ hz}) = 0.0063 \text{ inches}$ . This vertical rate varies with the **Cosine** of the **151.4 hertz** vibration phase angle:

$$\text{Cos}[(\pi/2) \cdot 0.707] = 0.444$$

During this interval  $t_R$ , the rear of the bullet contact patch could have been pushed lower by up to  $0.444 \cdot (6 \text{ ips}) \cdot 0.0000322 = 85.6 \text{ micro-inches}$  as the muzzle is sweeping downward at its sinusoidally varying rate during bullet exit. The bullet of radius **0.2551 inches** would then be given a “nose high” attitude  $\alpha$  of

$$\alpha = 0.0000856 / 0.2551 = 0.336 \text{ mrad (0.0192 degrees)}.$$

More importantly, since this forced attitude change took place in only **32.2  $\mu\text{sec}$** , the mechanically imparted pitch-up rate  $\alpha\text{-dot}$  is

$$\alpha\text{-dot} = 0.000336/0.0000322 = 10.4 \text{ radians per second.}$$

This yaw-rate  $\alpha\text{-dot}$  could also be expressed as **597 degrees per second**. For comparison, the coning rate  $\omega_2$  for a bullet initially coning at **30 hertz** is  $2\pi*30 = 188.5 \text{ radians (or 10,800 degrees) per second}$ .

Note that the mechanically produced yaw attitude  $\alpha$  and yaw attitude rate  $\alpha\text{-dot}$  will always be in the *same* vertical direction, either both nose upward or both nose downward, as determined by the timing of bullet exit.

Our example bullet moving at an average speed of **2730 fps** through the muzzle-blast exits the muzzle-blast shockwave after travelling about **7.5 inches** (15 calibers) from the muzzle and after flying for **229  $\mu\text{sec}$**  through the muzzle-blast zone. If this mechanically produced yaw-rate  $\alpha\text{-dot} = 597 \text{ degrees per second}$  simply continued unchanged throughout the muzzle-blast zone, the initial yaw  $\alpha_0$  of the bullet at the beginning of its ballistic flight would then be

$$\alpha_0 = 0.0192 + 597*0.000229 = 0.156 \text{ degrees,}$$

which would then have considerable aeroballistic effect, roughly equivalent to that of a **5 MPH** crosswind.

#### 4.4 Aerodynamic Effects in the Muzzle Blast Region

The base pressure behind the rifle bullet ranges from about 8,000 psi to about 18,000 psi on bullet exit from the muzzle, depending on the cartridge fired and the barrel length. These gasses are hot at about 6,000 degrees Fahrenheit and exit the muzzle behind the bullet at hypersonic initial airspeeds. Propellant gas exit pressures in this range allow muzzle-attached brakes to operate more or less effectively in reducing felt recoil. Here, with this 44-inch barrel length, the base pressure at bullet release is just **8400 psi**.

High speed video shows a “precursor” shockwave exiting the muzzle while the rifle bullet is still several bullet lengths deep in the bore behind the muzzle. This is the “air” in the bore ahead of the bullet being expelled into the undisturbed outside atmosphere by the fired bullet acting as a supersonic gas compressor piston. This precursor shockwave is propagating spherically at **Mach 1.0** in the same atmosphere through which

the bullet will be flying in subsequent ballistic flight. Absent any attached muzzle device, this shockwave propagates spherically from a virtual center on the axis of the bore about 2 calibers ahead of the muzzle. Being able to calculate the airspeed of this precursor shockwave in ambient air allows a handy distance/time metric in analyzing these videos.

After bullet release, the high-pressure, high-temperature gasses (and particles) exit the muzzle at a high enough rate so as mostly to fill the volume of the expanding spherical precursor shock. Their higher-pressure, hotter, higher-Mach-speed shockwave bubbles eventually merge with the expanding precursor shockwave just as the supersonic bullet pierces their combined shockwave to begin ballistic flight and terminate all muzzle-blast effects on the fired rifle bullet. This termination of these relatively low-pressure muzzle-blast effects occurs after about 15 calibers (7.5 inches for a 50-caliber bullet) of bullet travel, which requires **229  $\mu\text{sec}$**  of flight time at an average bullet speed of 2730 fps.

Aerodynamically, the groundspeed of the fired rifle bullet is increased by a few percentage points due to the hot gasses impacting the rear of the bullet. There is little point in calculating any reverse aerodynamic coefficient of drag for the bullet because this gas flow decays rapidly (approximately exponentially), and its maximum effective duration here is only about 229 microseconds.

A much more significant aerodynamic effect is the *amplification* of any mechanically produced initial yaw  $\alpha$  and yaw-rate  $\alpha\text{-dot}$  (both in the same vertical direction) caused by the reverse aerodynamic overturning moment impulse created by the high-speed gasses passing up the very slightly yawing fired bullet within the muzzle-blast zone. We know from many firing tests using indexed flat-base bullets which had been altered to have even very slightly canted bases (in the range of **0.10 degrees**), that after passing through the muzzle-blast zone, their subsequent ballistic flights show significant and predictable effects. It has been said that “a rifle bullet steers by its base.”

The reverse aerodynamic **lift force** acting through its reverse center of pressure ( $\text{CP}_R$ ) creates an overturning moment  $\mathbf{M}_R$  which acts directly to increase the magnitudes of any mechanically induced yaw attitude  $\alpha$  and yaw-rate  $\alpha\text{-dot}$ . These forces and moments are impulsive in nature. This



yaw disturbance occurs much too quickly for any gyroscopic effects to begin. The twist-rate of the rifling and the resulting spin-rate of the fired bullet simply do not matter here.

The increased yaw angle  $\alpha$ , in turn, additionally increases the lift force and its resulting overturning moment in an exponentially increasing spiral. However, the speed and density of the overtaking gasses are simultaneously decreasing, also about exponentially, thus reducing the reverse aerodynamic lift force being generated, so that the net effect is a finite amplification ratio  $R$  of the yaw attitude  $\alpha$  in any specific case. Not only is the **product**,  $\alpha \cdot R$ , likely to be large enough to cause significant subsequent aeroballistic effects, but the **sum** of their individual **variances** leads to accuracy issues and large shot-to-shot variability in air-drag (and in measured BC).

If the yaw attitude  $\alpha$  of the fired bullet after transiting the muzzle-blast zone is increased from its mechanically induced value of **0.156 degrees** to approximately **0.468 degrees** nose-high for our example bullet traversing the muzzle-blast zone (with  $R \approx 3.0$ ), this initial aeroballistic yaw attitude  $\alpha_0$  *would* be quite significant in subsequent ballistic flight, causing its own *horizontally rightward* aeroballistic jump trajectory deflection.

The reverse aerodynamic amplification of the bullet's yaw-rate  $\alpha\text{-dot}$  during passage through the muzzle-blast zone is even more significant for subsequent ballistic flight. It would be reasonable to assume that  $\alpha\text{-dot}$  might also increase by about this same (highly variable) amplification ratio  $R \approx 3.0$ , in which case

$$\alpha\text{-dot} \approx 10.4 \cdot 3.0 = 31.2 \text{ radians (1788 degrees)/second}$$

going into ballistic flight, or about **17-percent** of its likely initial coning rate.

#### 4.5 Bullet Design Considerations

Some rifle bullet designs will generate less reverse aerodynamic lift force, and hence less overturning moment, than other designs optimized only for minimum air-drag in forward ballistic flight. For example, a flat-based rifle bullet will have a significantly lower Coefficient of Lift in reverse flight for small yaw angles than would a boattailed bullet design, which would excel in forward aerodynamic flight. By shifting the CG of the rifle bullet design

further aft, nearer to the reverse center of pressure ( $CP_R$ ), the reverse aerodynamic overturning moment  $M_R$  can be reduced even more. On the other hand, a rounded convex-base bullet will have a significantly larger coefficient of lift in reverse aerodynamics at all yaw attitudes and would suffer greater reverse overturning moment while transiting the muzzle-blast zone. I have recently demonstrated this accuracy problem in test firing my own rounded base design copper Ultra-Low-Drag (ULD) bullets. I have now modified their boattails to a flat-base design with sharp rear corners.

## 5. Subsequent Ballistic Flight Effects

Entering into ballistic flight with significant vertically upward direction yaw attitude  $\alpha$  and yaw-rate  $\alpha\text{-dot}$  (in the same direction) can cause significant flight problems for the rifle bullet. If the initial yaw  $\alpha$  and yaw-rate  $\alpha\text{-dot}$  are vertically nose-upward, and the barrel is rifled in the usual right-hand twist direction, any horizontal crosswind blowing from *left-to-right* at the firing point would require the bullet to cone around the apparent wind direction with an unusually large initial coning angle  $\gamma$ .

The crosswind-induced yaw angle  $\gamma$ , the initial yaw angle  $\alpha_0$ , and the effect of the yaw-rate  $\alpha\text{-dot}$  all sum as magnitudes in determining the magnitude of the first maximum-yaw coning angle  $\alpha_{\text{Max}}$  at the beginning of ballistic flight. For direct 3:00 or 9:00 horizontal crosswinds, the crosswind angle of attack  $\gamma$  is

$$\gamma = \text{Tan}^{-1}[W/V] \approx W/V \quad (\text{for } W \ll V)$$

For a typical **10 MPH (14.7 fps)** crosswind,  $\gamma = \mathbf{0.308 \text{ degrees}}$  here.

As a metric concerning the effect of a non-zero initial yaw-rate  $\alpha\text{-dot}$ , we note that with  $\alpha\text{-dot} = \omega_2$  (the initial gyroscopic coning rate), there would be **no increase** in the coning angle  $\gamma$  needed for a favorable crosswind, and just **twice** the usual coning angle  $\gamma$  required to cone around an unfavored side crosswind ( $\alpha\text{-dot} = -\omega_2$ ) at the firing point.

Coning around the “eye” of any right-to-left crosswind would be unusually easy for this bullet, and a smaller initial coning angle  $\gamma$  would be needed. The bullet with  $\alpha\text{-dot} > 0$  (positive upward) is essentially already beginning

its clockwise coning motion about the eye of a right-to-left crosswind as it commences ballistic flight, but must overcome and reverse its initial direction of motion to begin coning about the leftward offset  $\gamma$  for any horizontal left-to-right crosswind.

A right-hand spinning bullet can only cone clockwise as seen from behind at its gyroscopically determined coning rate  $\omega_2$ , so to circle around an unfavorable crosswind approaching from its left side requires it first to circle far out rightward which could potentially *double* its initial coning angle for that left-to-right crosswind speed.

The yaw-drag penalty for flying with an angle of attack  $\alpha_{\text{Max}}$  is proportional to  $\delta^2 = \text{Sin}^2(\alpha_{\text{Max}}) \approx (\alpha_{\text{Max}})^2$ , so, since  $\alpha_{\text{Max}} = \gamma$  for any favorable direction crosswind, and  $\alpha_{\text{Max}} \leq 2*\gamma$  for an unfavorable direction crosswind, the yaw-drag penalty itself could be increased by a factor of up to **4** for any unfavorable crosswinds at the firing point.

This extra yaw-drag penalty with unfavored direction crosswinds at the firing point not only significantly reduces the effective BC of the fired bullet, increasing its time-of-flight to the target, but increases its drag-dependent leftward windage correction needed to stay centered on that target.

With a right-hand twist barrel, a vertically nose-up initial attitude  $\alpha$  produces its own aerodynamic jump trajectory deflection horizontally rightward during the first coning cycle of ballistic flight. This jump deflection early in flight, together with the long-range rightward spin-drift, offsets a portion of the rightward windage correction required with a favored right-to-left crosswind.

So, if you plan to shoot with rifle barrels which are too long to allow tuning out the bullet's initial pitch-up attitude with your selected bullets, perhaps you might wish to consider having ***matched pairs*** of rifle barrels made with ***opposite direction*** rifling twists. Since rifle barrels and their muzzle attachments using 60-degree V-threads need only about **30 foot-pounds** of installation torque, they are easily changed at the range based on the crosswind direction expected at the firing point.

Remember:

***“Use left-hand twists for left-side winds, and right-hand twist for right-side winds.”***