Bullet Obturation of Rifle Barrels

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Introduction

The sealing, or obturation, of the hot gasses produced during combustion of gunpowder propellants is a significant concern in interior ballistics. Leakage of some of the hot gasses past the projectile in the barrel causes damage to the barrel and to the projectile. This gas leakage is also responsible for much of the observed variation in muzzle velocities from one shot to the next. Highest speeds are measured for shots where barrel obturation was best, and reduced speeds are produced when propellant gasses blow by the bullet at peak chamber pressure and thereafter. Early gas leakage damages the sealing surfaces of the bullet and promotes continued gas leakage during the remainder of that bullet's trip through the rifle barrel.

Direct evidence of damage to a copper or copper-alloy jacketed bullets is shown by the uniform deposition of metallic copper inside the bore of a rifle barrel toward its muzzle. Elemental copper is vaporized by the hot powder gasses leaking past the bullet at peak base pressure. This hot copper vapor cools and expands ahead of the bullet and precipitates out on the cold steel surfaces of the barrel toward the muzzle. This copper is easily removed in routine barrel cleaning, as distinct from "copper fouling" which is the shearing of copper particles from the bullet due to rough interior surfaces in the throat or bore of the barrel.

Additional direct evidence of gas blow-by can be seen in high-speed video of the rifle firing. A dense cloud of smoke often obscures the emergence of the bullet itself from the muzzle.

One useful approach to improving barrel obturation during firing is to select barrels rifled in a way which improves this sealing of the powder gasses with any rifle bullet fired. Better obturation is supported by selecting rifling patterns where (1) the sides of the rifling lands are sloped significantly outward toward their bases, where (2) the bottom inside corners of the grooves are significantly radiused at their edges, and where (3) the top edges of the rifling lands are slightly radiused at both outside corners.

Boots Obermeyer's **5R** and Gary Schneider's **P5** rifling patterns come to mind as ideal rifling designs for promoting better bullet obturation.

Proper barrel obturation by the rifle bullet is a more complex problem than most riflemen realize. This is largely because the inside diameter (ID) of the 338-caliber rifle barrel has expanded by almost **0.001-inch** at the instant when the peak base pressure **P** is driving the bullet down the barrel. This is also the very time when gas obturation is most critical and most difficult for the bullet being fired to accomplish. We will show how this internal barrel expansion with internal pressure can be calculated accurately and how monolithic copper rifle bullets can be designed to deal with it.

Barrel ID Expansion with Internal Pressure

If we assume that a typical peak chamber pressure in firing 338-caliber rifle cartridges is **60 ksi** (thousands of pounds per square inch), we can use the interior ballistics program, QuickLOAD©, to estimate the peak base pressure **P** acting on the bullet, which will have travelled only **3.2 inches** down the barrel at that time, as **51 ksi** or **85 percent** of that peak chamber pressure. Our example Heavy Varmint profile 338-caliber match barrel has an outside diameter (**$2*r_o$**) of **1.24 inches** at that bullet location. We shall use the nominal groove diameter of **0.3380 inches** as the inside diameter (**$2*r_i$**) of this barrel.

Any rifle barrel qualifies mechanically as a thick-walled cylindrical pressure vessel which is not constrained in axial-direction expansion with increasing internal pressure. Fortunately, we have Lamé's Equations for calculating accurately the two-dimensional (radial and tangential) stresses and the amount of radial expansion **U(r)** for any point within the steel walls at radius **r ($r_i \le r \le r_o$)** from the axis of the rifle barrel as functions of the internal pressure **P** being applied. These equations hold for rifle barrels made of isotropic materials which have not been pre-stressed and which are not operated beyond their elastic limits.

The operative form of Lamé's Equation for finding the radial displacement **U(r)** for any location within a thick-walled cylindrical pressure vessels is:

$$U(r) = (P*r/E)*[(1 - \mu) + (1 + \mu)*(r_o/r)^2]/[(r_o/r_i)^2 - 1]$$

where

$U(r)$ = Radial expansion in inches at any radius r from axis
$r_i \le r \le r_o$
P = Internal pressure in psi = 51,000 psi here
E = Young's Modulus of Elasticity = 29,000,000 psi (steel)
r_o = Outside radius of cylinder = 0.620 inches here
r_i = Inside radius of cylinder = 0.169 inches here
μ = Poisson's Ratio = 0.30 for steel.

In particular, by setting **$r = r_i$**, the calculated value **$U(r_i)$** becomes the internal radial expansion of the ID of the pressurized barrel:

$U(r_i) = 0.000434$ inches

And the internal diameter (ID) expansion is **0.868 thousandths of an inch**.

While we are here, we might as well calculate the outside diameter (OD) expansion of the rifle barrel over this same part of the bore by setting **$r = r_o$**:

$U(r_o) = 0.000175$ inches

As expected, the OD expansion over this point is smaller at just **0.350 thousandths of an inch**. This explains how external strain gauges can allow laboratory measurement of base pressures behind the bullet up and down a test barrel.

Had our 338-caliber barrel been of much lighter profile with a **0.75-inch** OD at this point ahead of the chamber swell, for example, the internal diameter expansion would have been **1.076 thousandths of an inch**.

Note that these ID and OD expansions with internal pressure are independent of the quality, heat treatment, and strength ratings of the barrel steel. These calculated expansions are quite accurate provided the barrel material has never exceeded its yield stress, which would typically first occur in the steel strands immediately surrounding its bore.

Button rifling is a barrel rifling process which purposely *does* over-stress this internal steel and leaves behind residual implanted tangential (hoop) stresses which actually *do* reduce subsequent bore expansions with internal pressures somewhat. Without conducting a detailed analysis, we estimate that button-rifled barrels will expand internally about **2/3** as much as similar profile and caliber cut-rifled barrels at similar internal pressures.

Barrel Obturation with Jacketed Lead-Cored Rifle Bullets

Conventional jacketed, lead-cored rifle bullets typically obturate conventionally rifled barrels reasonably well, at least well enough for military and big game hunting rifle applications. We will discuss barrel obturation by a typical match-type jacketed rifle bullet made with a core material of essentially pure lead.

The soft lead core readily "slugs up" so that the bullet OD easily matches even the pressure-expanded ID of the steel rifle barrel at an assumed peak base pressure **P** of **51 ksi** driving the bullet forward. This permanent, plastic, bullet diameter enlargement is accompanied by a commensurate plastic foreshortening of the lead core within its jacket.

With a value of only **2,030,000 psi** for Young's Modulus of Elasticity for pure lead ($\mathbf{E_L}$), this lead core material has virtually no elastic "memory" of its pre-stressed shape. This is why lead is called a "dead metal." You deform it, and it just sits there waiting to be deformed again.

The radial contact pressure $\boldsymbol{\sigma_{rcp}}$ of the jacketed bullet against the steel walls of the barrel at peak base pressure **P** is

$$\boldsymbol{\sigma_{rcp} = \mu_L * \sigma_a = 0.44*51 \text{ ksi} = 22.44 \text{ ksi}}$$

which is well above the yield strength ($\mathbf{S_L}$) for pure lead of just **1740 psi**. This pure lead core material is acting almost as a liquid, at least as far as the transfer of pressures is concerned.

As the base pressure **P(t)** subsequently drops and the barrel begins returning to its unstressed ID, the contact pressure $\boldsymbol{\sigma_{rcp}}$ farther down the barrel becomes

$$\boldsymbol{\sigma_{rcp} = S_L + \mu_L * P(t) = 1.74 + 0.44*P(t) \text{ ksi}}$$

This peak $\boldsymbol{\sigma_{rcp}}$ value of **22.4 ksi** for a lead-cored rifle bullet gives us a good indication of how much radial contact pressure $\boldsymbol{\sigma_{rcp}}$ is required for any rifle bullet to seal **51 ksi** of gas pressure effectively within any rifle barrel. If $\boldsymbol{\sigma_{rcp}}$ were always to equal or exceed the base pressure **P(t)**, one might say that "perfect mechanical obturation" has been achieved between the two smooth surfaces. These copper-alloy jacketed, lead-cored rifle bullets have about 125 years of development and testing behind them.

<u>Barrel Obturation with Copper Bullets</u>

This paper addresses the design of monolithic copper bullets so as to achieve ***effective obturation*** of the hot powder gasses throughout the interior ballistics portion of their firing process. A 338-caliber copper ULD bullet of my own design is used as an illustrative example.

Some monolithic bullet designs utilize a sequence of over-diameter gas sealing rings which are designed always to be plastically compressed during bullet engravement. A similar approach is used in artillery shell designs. Plastic deformation in compression always includes a *maximum elastic compression* at the elastic limit for the material of the sealing ring. A down side of using multiple small gas sealing rings for monolithic copper rifle bullets is the necessarily higher aeroballistic drag induced by the multitude of secondary shock waves which these sealing rings invariably throw off during supersonic and transonic flight. Another disadvantage of using narrow sealing rings for barrel obturation is that their elastic “working length” in providing contact pressure is only the compressed height of the sealing rings themselves, typically just a few thousandths of an inch, as opposed to the full radius of an engraved bullet shank or wide driving/sealing band. These are among the reasons we selected a rear driving/sealing band design for our monolithic copper ULD bullet designs.

Understanding basic physical concepts can increase our understanding of what happens to a monolithic copper rifle bullet in the interior ballistics phase of rifle firing. Inside the rifle barrel, the copper bullet does not act entirely as a “rigid body” as it does for all practical purposes in exterior ballistics. The copper bullet material is subjected to forces and pressures large enough to cause significant elastic and some plastic deformations in its shape. We need to understand how these bullet distortions might affect the ability of the copper bullet to seal, or obturate, the hot powder gasses most effectively and thereby minimize any damaging gas blowby during firing.

Let us start by looking at the rifling engraved bullet at the moment of peak chamber pressure (say **60.0 ksi**) which occurs when the bullet has travelled just a few inches down the rifle barrel (**3.2 inches** here) from its initial position. According to the interior ballistics program *QuickLOAD©* the peak base pressure **P** accelerating our example 240-grain copper 338 bullet is **85 percent** of the peak chamber pressure at this point so that **P = 51.0 ksi** in this example. The base pressure **P** gradually becomes a smaller fraction (**82.5 percent** at the muzzle) of the instantaneous chamber pressure as the bullet speed increases and as it travels farther down the somewhat gas-flow restrictive barrel. Base pressure behind the bullet is only **10.8 ksi** at the muzzle of our example **28-inch** barrel.

The base pressure **P** exerts a distributed force **F** on the bullet accelerating its entire mass forward. The area **A** over which this force **F** is distributed can best be thought of as that of the rearmost barrel-obturating aperture area, an imaginary plane within the rearmost full diameter portion of the rifle bullet. This plane area **A** is also ideally equal to the cross-sectional area of the rifled bore of the obturated barrel. The bullet material in front of this moving obturation aperture is being shoved forward by the distributed force **F**, while the afterbody (boat-tail) material of the monolithic bullet behind this plane is simply being dragged along via its mechanical attachment to the shank of the bullet.

At any instant, we can reason that, hydrostatically speaking,

$$\mathbf{F = P*A}$$

That distributed force **F** produces an axial-direction stress $\boldsymbol{\sigma_a}$ on the copper bullet material within this obturating aperture given by

$$\boldsymbol{\sigma_a} \mathbf{= F/A = P = 51.0\ ksi}$$

Note that this axial stress $\boldsymbol{\sigma_a}$ depends only on the base pressure **P** and is independent of the caliber of the bullet.

If that axial stress $\boldsymbol{\sigma_a}$ does not exceed the elastic limit **S** = **40,000 psi** for this “half hard” copper material, it will produce an axial, compressive elastic strain ratio $\boldsymbol{\varepsilon_a}$ in this bullet material given, in accordance with Hooke’s Law, as

$$\boldsymbol{\varepsilon_a = \sigma_a} \mathbf{/E = P/E = 0.003018}$$

where **E = 16,900,000 psi** is Young’s Modulus of Elasticity for this copper bullet material. While the bullet material within the obturating aperture is stressed somewhat beyond its elastic limit **S** in this example, it is confined within the steel barrel and has nowhere else to go.

We also know that this compressive axial stress $\boldsymbol{\sigma_a}$ would, if not constrained by the steel walls of the barrel, produce a radial elastic strain $\boldsymbol{\varepsilon_r}$ on this same bullet material within the obturation aperture given by

$$\varepsilon_r = \mu * \sigma_a / E = \mu * P / E = 0.000996$$

where **μ = 0.33** is Poisson's Ratio of shrinkage to elongation during tensile and compressive testing of this copper material.

We can see why this is so if we consider in isolation the thin disc of copper material within the obturating aperture. Unstressed, this thin disc has a radius **r** and a thickness of **h**. Under axial compressive stress $\boldsymbol{\sigma_a}$ within its elastic range, the thickness of the disc is reduced to **h – dh**, and its radius would increase to **r + dr**, being unconstrained here.

If the unstressed *volume* of this disc, **A*h**, were to remain *constant* under this axial stress $\boldsymbol{\sigma_a}$, we could say

$$\pi * r^2 * h = \pi * (r + dr)^2 * (h - dh)$$

Dividing through by $\boldsymbol{\pi * r^2 * (h - dh)}$, and neglecting high-order terms, this expression simplifies to

$$(dr)/r = 0.5 * (dh)/h$$

or, in terms of radial and axial elastic strain ratios

$$\varepsilon_r = 0.5 * \varepsilon_a$$

This "constant volume" condition actually holds true only for an "incompressible" liquid such as water in low-pressure hydrostatics. It is almost statically true for a "perfectly elastic" material such as soft gum rubber which has a value approaching **0.5** for Poisson's Ratio (**μ**). Here, we must replace the value **0.5** with **0.33**, Poisson's Ratio (**μ**) for copper:

$$\varepsilon_r = \mu * \varepsilon_a = \mu * \sigma_a / E = \mu * P / E$$

Or, multiplying through by Young's Modulus **E**, we find the radial stress $\boldsymbol{\sigma_{rbp}}$ caused by the base pressure **P** acting axially to be

$$\sigma_{rbp} = \mu * \sigma_a = \mu * P = 16.8 \text{ ksi}$$

At least we now see why Poisson's Ratio **μ** can never exceed **0.5** for any solid material which retains internally a portion of any cross-axis stress applied to it. The ability of a metal object to retain stress internally allows it to retain a “memory” of its pre-stressed shape.

Now, let us consider what happens when this unstressed copper aperture sealing disc *exactly fits* the interior of the rifle barrel at this point where maximum base pressure **P** is to be applied to it. That is, let us assume for the moment that it has **zero** unloaded radial contact pressure σ_{r0}. Let us also assume for the moment here that the much stronger steel walls of the rifle barrel do not move outward with these interior pressure stresses.

When the base pressure **P** is applied, the radial contact pressure σ_{rcp} is

$$\sigma_{rcp} = \sigma_{r0} + \sigma_{rbp} = 0 + \mu{*}P = 16.8 \text{ ksi}$$

Perhaps we could achieve more perfect obturation by starting with a non-zero static contact pressure σ_{r0}. We could have radially compressed an over-diameter copper bullet in the throat of the barrel by an amount Δr so that its static contact pressure σ_{r0} is

$$\sigma_{r0} = E{*}(\Delta r)/r \le S$$

where **S = 40,000 psi** is the rated yield strength of the half hard copper bullet material.

One should design the maximum outside diameter (OD) of the bullet shank or its driving/sealing band so that, within bullet manufacturing tolerances, the bullet OD will always be at least as large as the maximum groove inside diameter (ID) for standard specification rifle barrels of each caliber. With a bullet diameter production tolerance of **+/- 0.0002 inch** (or even less), we specify a rear driving/sealing band OD **0.0006-inch** larger than the **nominal groove ID** for standard barrels of that caliber. Then, when specifying the chambering reamer design, we specify a ball seat inside diameter **0.0008-inch** larger than this nominal groove ID to minimize gas blowby before the bullet is engraved by the rifling. Match grade barrel production groove ID tolerance can be specified to fall between the **nominal groove ID** and the **nominal groove ID+0.0002 inch**, with less than **0.0001-inch** variation, end to end. Thus, any production bullet within specifications will freely enter the ball seat and statically seal the fitted production barrel blank during firing.

For our 338-caliber copper bullet, **Δr** is nominally **0.0003 inch**, and **r** is **0.1690 inch**

$$(\Delta r)/r = \varepsilon_{r0} = (0.0003/0.1690) = 0.001775 < \varepsilon_{Max}$$

The “half hard” copper from which these bullets are manufactured has a minimum yield strength **S** rating of **40,000 psi** and a Modulus of Elasticity **E** of **16,900,000 psi**. Thus, its maximum possible elastic strain ratio ε_{Max} is given by

$$\varepsilon_{Max} = S/E = 40{,}000/16{,}900{,}000 = 0.002367$$

The selected nominal radial compression **Δr = 0.0003 inches** produces ¾ of the *maximum* radial stress pre-load

$$\sigma_{r0} = E*(\Delta r)/r = 30.0 \text{ ksi}$$

If the production and wear tolerances stack so that **Δr** has its minimum value of **0.00015 inches**, $\sigma_{r0} = 15.0$ **ksi**.

If the production and wear tolerances stack so that **Δr** has its maximum value of **0.00045 inches**, $\sigma_{r0} = 40.0$ **ksi**, still, and the rear driving/sealing band of the copper bullet is permanently compressed (plastically) by **0.00005 inches** in diameter.

The total radial contact pressure now looks pretty good at a nominal

$$\sigma_{rcp} = \sigma_{r0} + \sigma_{rbp} = 30{,}000 + 16{,}800 = 46.8 \text{ ksi}$$

which is almost the base pressure **P = 51.0 ksi**, and would produce almost “perfect mechanical obturation” as mentioned earlier.

But we have to go back and account for the steel walls of the barrel expanding by **Δr = -0.000434 inches**. The change σ_{rexp} in copper radial bearing stress due to internal expansion of the steel barrel would be given by

$$\sigma_{rexp} = E*(\Delta r)/r = 16{,}900{,}000*(-.000434/0.1690) = -43.4 \text{ ksi}$$

However, $\sigma_{rexp} = 40.0$ **ksi**, because more than the maximum implantable stress cannot be removed by relaxing the constraint on bullet OD.

And the total radial bearing stress σ_{rcp} of the copper bullet inside the expanded barrel is now given by

$$\sigma_{rcp} = \sigma_{r0} + \sigma_{rbp} + \sigma_{rexp} = 46{,}800\text{-}40{,}000 = 6.8 \text{ ksi}$$

This is now only about **30-percent** of the gas sealing pressure **22.44 ksi** of the lead-cored bullet studied earlier and likely will not seal the gasses effectively. There is another way to increase this radial bearing stress for monolithic copper bullets, and that is by drilling their bases axially to port the base pressure inside the gas sealing portion of those bullets.

Base Pressure Ducting by Base Drilling of Copper Bullets

Large potential (unconstrained) diameter increases and corresponding (constrained) increases in radial surface contact pressures are available by porting the base pressure **P** forward into the body of the monolithic copper bullet. To be effective in improving obturation, the base drilling depth needs to pass completely through the boat-tail and at least mostly through under the rear driving/sealing band.

Test firings of 338-caliber copper ULD bullets show that a base-drill diameter of **0.166-inch** allows the copper rear driving band to expand temporarily in outside diameter during firing at **60.0 ksi** peak chamber pressure by enough to ***obturate completely*** in any reasonable-sized rifling grooves. Unusually high muzzle velocities and single-digit velocity spreads were also obtained in these firing tests. The desired amount of maximum radial OD expansion (**$2*Ur_o$**) can be varied simply by adjusting the base-drill diameter (**$2*r_i$**) slightly.

The operative form of Lamé's Equations for thick-walled cylindrical pressure vessels (here the rear driving/sealing band of our copper bullet) is:

$$U(r) = (P*r/E)*[(1 - \mu) + (1 + \mu)*(r_o/r)^2]/[(r_o/r_i)^2 - 1]$$

where

U(r) = Radial expansion at radius r from axis of cylinder
$r_i \leq r \leq r_o$
P = Internal pressure in psi = 54,000 psi here
E = Young's Modulus of Elasticity (Cu) = 16,900,000 psi
r_o = Outside radius of cylinder = 0.1693 inches here
r_i = Inside radius of cylinder = 0.0830 inches here
μ = Poisson's Ratio = 0.33 for Cu.

Here, we are calculating the maximum temporary elastic radial expansion **U** of the rear driving band of our 338-caliber copper ULD bullet as a function of radius **r** from the cylinder axis for $r_i \le r \le r_o$. In particular, we want to find the potential unconstrained radial expansion at the outside diameter. For this special case of $r = r_o$, Lamé's Equation above reduces to:

$$U(r_o) = (2*P*r_o)/\{E*[(r_o/r_i)^2 - 1]\}$$

Substituting our numerical values for this copper 338-caliber ULD bullet, we have:

$$U(0.1694) = (2*51{,}000*0.1693)/(16{,}900{,}000*3.1606)$$
$$U(0.1694) = 0.0003233 \text{ inches}$$

Thus, the outside *diameter* of the rear driving band could temporarily increases by a calculated **0.000647 inches** when a hydrostatic pressure of **51.0 ksi** is applied to the inside of the obturating surface of the hollow-base copper bullet. This radial expansion is *purely elastic* because **$U(r_o)$ = 0.0003233 inches** is less that the maximum elastic radial expansion of **0.0004007 inches** for these half hard copper bullets. When this internal pressure drops to **zero** during subsequent aeroballistic flight, the bullet returns to its (engraved) original shape.

This potential temporary diameter increase significantly improves the obturation of the monolithic copper ULD bullet ***exactly when it is most needed***. Fired test bullets recovered from the waters of a swimming pool show ***perfect obturation*** of these base-drilled bullets forward to the shoulder depth of the internal drilling. Other than the small bullet weight reduction penalty in ballistic coefficient (BC) involved, no other ill effects of this base-drilling upon bullet behavior could be observed. Base drilling also improves the gyroscopic stability of the copper bullets.

Now, if we constrain this potential bullet OD expansion due to base drilling to exert instead a radial pressure σ_{rbd} against the inside of the barrel walls, with **Δr = 0.0003233 inches**, this radial stress is given by

$$\sigma_{rbd} = E*(\Delta r)/r = 16{,}900{,}000*(.0003233/0.1693) = 32.3 \text{ ksi}$$

If we reduce the base-drill diameter from **0.166-inch** to **0.125-inch**, for example, the expression **$[(r_o/r_i)^2 – 1]$** in the denominator above increases from **3.1606** to **6.3376**, just about cutting the potential diametral expansion in half, from **0.647 thousandths** to **0.3225 thousandths of an inch.**

Correspondingly, with **$\Delta r = 0.0001612$ inches** with the smaller base drill,

$$\sigma_{rbd} = E^*(\Delta r)/r = 16.1 \text{ ksi}$$

The timing of this base-pressure ducting bullet expansion is the same as that of the inertial force driven expansion, a reduced amplitude and slightly delayed version of the chamber pressure curve.

With the **0.166-inch** drill size, the total copper bullet radial contact pressure **σ_{rcp}** is

$$\sigma_{rcp} = 6{,}800 + 32{,}300 = 39.1 \text{ ksi}$$

and with only a **0.125-inch** drill size, it is

$$\sigma_{rcp} = 6{,}800 + 16{,}100 = 22.9 \text{ ksi}$$

Thus, the **0.125-inch** base drill size looks about right for this 338-caliber, and any larger caliber, copper bullet. However, we must scale down this base drill diameter for smaller-caliber bullets to minimize weight reduction.

Bullet Expansion due to Centripetal Force

The instantaneous distributed centripetal force **df** acting outward on a thin cylindrical shell element of mass **dm** of the rear driving/sealing band, at radius **r** from the spin-axis of the bullet, is given by

$$df = dm^*r^*\omega^2$$

where **ω** is the instantaneous spin-rate of the bullet in radians per second as the bullet is spinning up while traversing the rifled barrel.

The mass element **dm** of the cylindrical shell can be formulated as

$$dm = \rho^*L^*(2\pi^*r)^*dr$$

where **L** is the axial length of the rear driving/sealing band.

Combining these expressions we have

$$df = 2\pi * \rho * L * \omega^2 * r^2 * dr$$

Integrating over the radius **r** from **zero** to **R = 0.1694 inches** to find the total outward-acting centripetal force **F** exerted by a 338-caliber copper bullet upon the constraining steel walls of the barrel, we have

$$F = (2/3) * \pi * \rho * L * \omega^2 * R^3$$

The outward stress **σ = F/A**, where the distributed working area **A = 2π*R*L**, so

$$\sigma = \rho * \omega^2 * R^2 / (3 * 12 * g)$$

The last two factors in the denominator are necessary if we want to give our copper density **ρ** as **2235.6/7000 = 0.31937 pounds per cubic inch** instead of using proper density units such as slugs per cubic foot. The acceleration of gravity **g** is taken to be **32.174 feet per second squared**.

Evaluating the strain ratio **ε = σ/E** at **R = 0.1693 inches** at the outer surface of the bullet, we have

$$\varepsilon = 4.6764 * 10^{-13} * \omega^2$$

The spin rate **ω** of the bullet at the muzzle would be **6600 revolutions per second** for our example bullet fired at **3300 fps** from a barrel rifled at **6 inches per turn**. Thus, at the muzzle **ω = 2π*6600 radians per second**, and the maximum strain ratio **ε** is

$$\varepsilon = 0.0008042$$

If unconstrained, the diameter enlargement due to centripetal force would then be **0.2723 thousandths of an inch**, which is small, but not completely insignificant, occurring as a constrained radial stress near the muzzle end of the barrel and as an unconstrained bullet diameter enlargement during subsequent free flight.

Even at **6600 revolutions per second**, the radial stress due to centripetal force σ_{rcf},

$$\sigma_{rcf} = 0.0008042 * E = 13.6 \text{ ksi}$$

is still far less than the yield strength **40.0 ksi** of this half hard copper bullet material.

This centripetal enlargement in diameter of monolithic copper bullets varies with the square of bullet spin-rate, which in turn varies linearly with bullet speed down the bore. It starts at zero, has negligible effect at the time of peak chamber pressure, but peaks rapidly as the bullet nears the muzzle, just as the inertial enlargement and base pressure ducting enlargement are reducing monotonically to their post-peak minimums. So their total combined effect varies slightly less with bullet position than with any single effect considered separately.

<u>Summary</u>

We can now see the total combined radial bearing pressure $\boldsymbol{\sigma_{rcp}}$ of the copper bullet sealing against the inside steel surfaces of the barrel at peak base pressure and beyond to be the sum, according to the Principle of Superposition, of five different indepently analyzed effects:

$$\boldsymbol{\sigma_{rcp} = \sigma_{r0} + \sigma_{rbp} + \sigma_{rexp} + \sigma_{rbd} + \sigma_{rcf}}$$

These five different contact pressure effects are:

(1) The contact pressure due to initial compression of the **0.0006-inch** over-diameter copper bullet

$$\boldsymbol{\sigma_{r0} = (0.0006/0.3380)*E = 30.0\ ksi}$$

[This contact pressure is modeled here as constant after rifling engravement.]

(2) The dynamic radial stress due to axial stress $\boldsymbol{\sigma_a}$ at **P = 51.0 ksi**

$$\boldsymbol{\sigma_{rbp} = 0.33*51000 = 16.8\ ksi}$$

(3) The loss in copper bearing stress due to barrel expansion by
Δr = -0.000434 inches

$$\boldsymbol{\sigma_{rexp} = -40.0\ ksi}$$

[For a pre-stressed button-rifled barrel, we estimate **Δr ≈ -0.0003 inches** with a corresponding contact pressure loss at this barrel expansion of only **30 ksi**.]

(4) The radial stress due to ducting of the base pressure **P** internally by axially drilling the bullet base with a **0.125-inch** diameter drill

$$\sigma_{rbd} = 16.1 \text{ ksi}$$

[The radial stresses σ_{rbp}, σ_{rexp}, and σ_{rbd} each have the timing of the base pressure **P** acting upon the bullet; that is, a reduced and delayed version of the chamber pressure curve.]

(5) The radial stress caused by centrifugal force as the bullet spins up

$$\sigma_{rcf} = 13.7 \text{ ksi}$$

at the full rotation rate of the bullet near the muzzle.

[The centrifugal force is negligible at the time of peak base pressure.]

Thus, the total radial contact pressure σ_{rcp} at the critical time of peak base pressure **P** sums to

$$\sigma_{rcp} = \sigma_{r0} + \sigma_{rbp} + \sigma_{rexp} + \sigma_{rbd} + \sigma_{rcf}$$

$$\sigma_{rcp} = 30.0 + 16.8 - 40.0 + 16.1 + 0.0 = 22.9 \text{ ksi}$$

This contact pressure matches that of a soft lead-core match bullet (**22.4 ksi**) fired under the same conditions. Thus, this base-drilled copper bullet should obturate the rifle barrel just as well.

While study of the elasticity of materials is more complex than this discussion would indicate, this elementary analysis is sufficient for our purposes here.

Conclusions

Copper rifle bullets can be made to seal the hot powder gasses as well or better than traditional jacketed, lead-cored bullets, but they have to be carefully designed and manufactured to enable them to do so. By holding the radial contact pressure of copper bullets (**22.9 ksi**) to just exceeding that of corresponding lead-cored bullets (**22.4 ksi**), barrel friction and fouling characteristics should remain quite similar.