

Calculating Yaw of Repose and Spin Drift (short form)

-A novel and practical approach for computing the Spin Drift perturbation-

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Introduction

The Yaw of Repose angle β_R is a very small, but gradually increasing, horizontally rightward, aircraft-type yaw-attitude bias or “side-slip” angle of the *coning axis* of a right-hand spinning bullet. The Yaw of Repose reverses sign and angles leftward for a left-hand spinning bullet.

We discuss only right-hand spinning bullets here for clarity. It can be shown that for right-hand twist, the yaw of repose lies to the right of the trajectory. Thus the bullet cones around with an average attitude offset to the right, leading to increasing side drift to the right.

For spin-stabilized bullets, this attitude angle creates the well known horizontally rightward Spin Drift displacements on long-range targets. The small horizontally rightward Yaw of Repose angle causes a small rightward aerodynamic lift force which, in turn, causes a slowly increasing horizontal velocity of the bullet.

It is important to realize that this effect occurs independently of the presence of surface wind of any force or from any direction. Bear in mind that the Yaw of Repose represents the horizontal yaw attitude of the bullet's *coning axis* or the average yaw of the coning bullet.

The acceleration of gravity acting upon the flat-fired bullet in free flight is the original cause of this small yaw-attitude bias angle. The downward curving of the trajectory due to gravity causes the airstream passing over the bullet to approach from below the nose of that bullet.

This wind shift during each coning cycle causes an increased aerodynamic angle of attack which peaks when the center of gravity (CG) of the bullet is at the Bottom Dead Center (BDC) position in each coning cycle where its nose is oriented maximally upward.

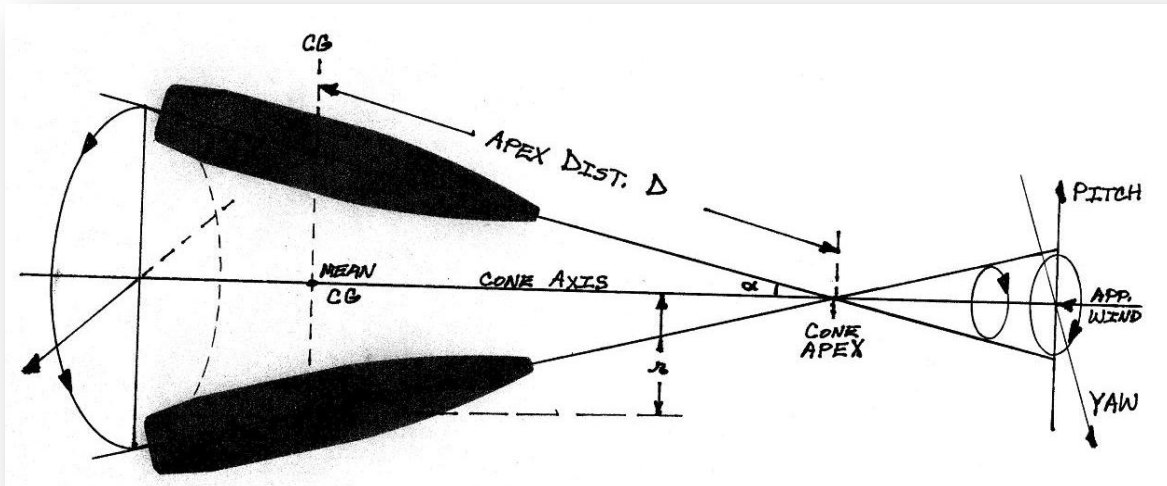


FIGURE: Extreme TDC and BDC Positions of the Coning Bullet

Since the coning angle α always exceeds this small change in the approaching airstream direction during each coning cycle, the aerodynamic angle of attack for a bullet at Top Dead Center (TDC) when the bullet's nose is pointing maximally downward is at a minimum for that coning cycle.

These modulations of aerodynamic angle of attack during each coning cycle produce small differential rightward-acting increments in the aerodynamic overturning moment experienced by the bullet centered upon the BDC and TDC positions of the bullet during each coning cycle.

In physics these are termed “torque impulses,” and each one pushes the angular momentum vector of the right-hand spinning bullet horizontally rightward without affecting its spin-rate. Torqueing the angular momentum vector of the spinning bullet rightward is tantamount to turning the nose of the flying bullet rightward.

Both the overturning moment vector \mathbf{M} and its differential torque impulse vector $\Delta\mathbf{M}$ point rightward for the bullet at its BDC location. While the vector \mathbf{M} itself points leftward as seen from behind when the bullet is located at TDC in its coning cycle, the *reduced* angle of attack at that position produces a *negative* differential vector $\Delta\mathbf{M}$ which is *positive rightward* at TDC as well.

The 175.16-grain 30-caliber US Army M118LR bullet used as an example here is experiencing its 88th half-coning cycle when it reaches the target distance of 1,000 yards. The reinforcing cumulative effect of this chain of rightward torque impulses occurring twice per coning cycle (or once each half cycle) is the mechanism by which gravity causes the slowly increasing rightward Yaw of Repose attitude bias of the flying bullet.

The horizontal spin-drift **SD** which we observe in long-range shooting is due to a horizontally acting aerodynamic *lift force* attributable to the small, but steadily increasing, Yaw of Repose attitude angle β_R of the coning rifle bullet.

We will use the principles of linear aeroballistics in formulating the Yaw of Repose and its resulting Spin-Drift.

Detailed analyses of PRODAS 6-DoF simulation runs show that in flat firing the magnitude of the spin-drift **SD** in any given simulated firing is, beyond the first 150 yards or so, *very nearly equal* to some invariant Scale Factor **ScF** of about **1.2 to 2.4 percent**, more or less, times the bullet's *drop from the projected bore axis*:

$$\mathbf{SD}(t) = \mathbf{ScF} * \mathbf{DROP}(t) \quad (1)$$

In other words, the horizontal spin-drift trajectory looks just like a small fraction **ScF** of the vertical trajectory rotated 90 degrees about the extended axis of the bore with both curvatures ultimately caused by the same gravitational effect.

We must formulate the Scale Factor **ScF** so that it can be accurately evaluated for any given bullet type and firing conditions.

After calculating the Scale Factor **ScF**, we need only an accurate determination of the bullet's *drop from the bore axis at the target distance* to calculate an accurate Spin-Drift at the target.

Existing 3-DoF trajectory programs specialize in the accurate calculation of this bullet drop at the target distance in any firing conditions.

In this paper we will use as our example bullet the 30-caliber 175-grain “M118LR” bullet as loaded in the US Army’s M118LR Special Ball (7.62x51 mm NATO) long-range sniper and match ammunition. We do this because we have several PRODAS 6-DoF simulation runs on hand (from 2011) for this 7.62 mm NATO ammunition, reporting the linear ballistic results (including Spin-Drift) for each millisecond of its **1.6923-second** total simulated flight time to **1000 yards**.

The bullet weight actually used in these PRODAS runs is **175.16 grains**. The simulated firing conditions are 1) flat firing, 2) standard sea-level ICAO atmosphere, 3) no wind, 4) no Coliolis effect calculated, 5) muzzle velocity of **2600.07 feet per second**, and 6) barrel twist is right-handed at **11.5 inches per turn**.

The “no-wind” and “no-Coliolis” conditions assure that the rightward spin-drift **SD** is the only secular horizontal “bullet drift” being computed by PRODAS. However, the PRODAS reported drift data necessarily includes the oscillating horizontal component of the bullet’s helical coning motion about its mean trajectory throughout its simulated flight.

We also have PRODAS runs available for this same bullet fired through constant left and right 10 MPH crosswinds as well as left-hand twist runs in each of the three constant wind conditions.

Estimating the Yaw of Repose

Based upon the above gravitational explanation, we have developed an integral expression for the Yaw of Repose angle $\beta_R(t)$ at any flight time t which well matches the available PRODAS data for this example M118LR bullet:

$$\beta_R(t) = (2\pi * g / t) \int [\omega_2(t) * V(t)]^{-1} dt \quad (2)$$

where $g = 32.174 \text{ feet/second}^2$ is the acceleration of gravity, $\omega_2(t)$ is the instantaneous coning (or gyroscopic precession) rate of the bullet in radians per second, and $V(t)$ is the instantaneous forward velocity of the bullet in feet per second.

In the absence of having 6-DoF simulation data available, we could approximate the Yaw of Repose angle $\beta_R(t)$ by assigning (closed form) integrable continuous functions of time t to represent the variables $\omega_2(t)$ and $V(t)$ so that we could then approximate this *summing* operation by performing the definite integration.

If we calculate the values of $\omega_2(t)$ and $V(t)$ at $t = 0$ and at a much later flight time $t = T$, and we assume for approximation purposes that each function decays exponentially with time t , then the definite integral for $\beta_R(t)$ can be expressed as:

$$\beta_R(t) = \{2\pi * g / [\omega_2(0) * V(0) * T]\} \int \exp[-(k\omega + k_v) * t / T] dt \quad (3)$$

with

$$k\omega = \ln[(\omega_2(T) / \omega_2(0))]$$

$$\omega_2(t) = \omega_2(0) * \exp[k\omega * t / T] \quad (4)$$

$$k_v = \ln[V(T) / V(0)]$$

$$V(t) = V(0) * \exp[k_v * t / T] \quad (5)$$

Here we are using t/T as a dimensionless canonical variable in the exponential decay expressions and as a dummy variable in the (summing) integration.

After the definite integration from $t = 0$ to $t = T$, the expression for $\beta_R(t)$ is:

$$\beta_R(t) = \{-2\pi * g / [\omega_2(0) * V(0) * (k\omega + k_v)]\} * \{\exp[-(k\omega + k_v) * t / T] - 1\} \quad (6)$$

Our “no wind” PRODAS runs for this M118LR bullet show that $V(t)$ slows from an initial velocity of **2600.07 feet per second** to **1340 FPS** (Mach 1.20) at **888.5 yards** downrange with a time-of-flight (T) of **1.430 seconds**, and the coning rate $\omega_2(t)$ of the bullet slows from **$2\pi * 45.57$ radians per second** to **$2\pi * 17.00$ radians per second** over this same interval of T seconds.

The Yaw of Repose $\beta_R(T)$ at $T = 1.430$ seconds, and at **888.5 yards** downrange, would then be calculated as:

$$k\omega = \ln[(\omega_2(T)/\omega_2(0))] = -0.98604$$

$$k_V = \ln[V(T)/V(0)] = -0.66328$$

$$\beta_R(1.430 \text{ sec}) = [1.6464 \times 10^{-4}] * [4.20344] = 0.69206 \text{ mrad} = 0.039652 \text{ degrees} \quad (7)$$

While PRODAS does not directly report β_R , this small **0.040-degree** angle is not unreasonable for this M118LR bullet at **888.5 yards** downrange, and it matches the horizontal deviation in the trajectory simulation quite well.

We will use this closed-form algorithm (**Eq. 6**) for estimating the Yaw of Repose angle $\beta_R(t)$ without relying upon 6-DoF simulation data in formulating the spin-drift **SD(t)** of any rifle bullet at long ranges.

Analytic Calculation of the Spin Drift at the Target

If we formulate a reliable estimation of the Scale Factor **ScF** for any given bullet in any given firing conditions, this Scale Factor **ScF** can then be used together with a reliably calculated value of that bullet's *DROP from the bore axis at the target* to calculate analytically the spin-drift **SD(t)** at the target for any given rifle bullet in any firing conditions according to **Eq. 1**:

$$\text{SD}(t) = \text{ScF} * \text{DROP}(t)$$

We know from examination of available “constant crosswind” PRODAS 6-DoF simulations that the Scale Factor **ScF** needs to be **0.0219685** for the “constant no-wind” runs, and **0.0222219 (or 1.154 percent larger)** for the “constant 10 MPH crosswind” runs for this example M118LR bullet fired in these simulated conditions to **1,000 yards**.

The constant “no-wind” case represents the *minimum possible SD(t)* values for this bullet fired at this muzzle velocity and spin-rate in this atmosphere and flying with the *minimum possible coning motion* throughout its flight.

Most *dynamically stable* rifle bullets fired outdoors at long ranges will likely suffer only the minimal **1.154 percent** increased “constant 10 MPH crosswind” type of spin-drift **SD(t)** at long ranges.

However, a *dynamically unstable* rifle bullet, such as the infamous mid-range 30-caliber 168-grain Sierra International, might experience about **5 percent** greater spin-drift (**SD**) when fired to long ranges (up to 800 meters) through *any* non-zero crosswinds. The PRODAS data show that the M118LR bullet is considered to be dynamically stable in supersonic flight.

Estimating the Spin Drift Scale Factor - ScF

We formulate an estimator for an **ScF** value of the “constant 10 MPH crosswind” type of coning motion for our example bullet which can be duplicated for any other *dynamically stable* bullet in any likely firing conditions.

In flat firing, we can formulate this Scale Factor **ScF** in terms of the *ratio* of 1) the horizontal aerodynamic lift-force acting on the flying bullet due to its Yaw of Repose β_R as that bullet nears its long-range target to 2) the vertically downward-acting weight of that bullet. In this manner, we can formulate **ScF** for any given bullet and likely wind conditions as:

$$\text{ScF} = 1.01154 * 0.383703 * [q(t) * S] * \sin[\beta_R(t)] * CL_{\beta}(t) / Wt$$

$$\text{ScF} = 0.388132 * [q(t) * S] * \beta_R(t) * CL_{\beta}(t) / Wt \quad (8)$$

with **Wt** = 175.16/7000 representing the weight of this bullet given in pounds-force **lbf**.

The initial constants (**0.383703** and **0.388132**) have been empirically determined from several PRODAS runs and should be the same for any *dynamically stable* rifle bullet in any firing conditions likely to be encountered in long-range rifle shooting.

These PRODAS simulations, together with the classic formulation for the yaw of repose angle $\beta_R(t)$, indicate that the Scale Factor **ScF** might need to be increased by about **5 percent** for *marginally dynamically unstable* bullets which will fly with significant coning angles throughout their flight when fired through *any non-zero crosswinds*.

Each of these functions of time **t** should be evaluated at the time **T** when the bullet has slowed to an airspeed of **1340 feet per second**, or approximately **Mach 1.20**, depending upon ambient conditions.

This flight time **T** and the flight distance at which it occurs are completely independent of the actual range to the target. The time **T** can be determined by using a 3-DoF point-mass trajectory calculator.

Any good long-range rifle bullet should remain safely above the turbulent transonic region at this **1340 fps** airspeed in almost all atmospheric conditions.

The more “aerodynamic” of our lowest-drag long-range rifle bullet designs do not encounter transonic buffeting until they slow to about **Mach 1.10** airspeed. The needed coefficient of lift CL_{β} is particularly difficult to estimate for any bullet at transonic airspeeds.

Most experienced long-range riflemen select their shooting equipment so that whenever possible their bullets impact the target at airspeeds above **Mach 1.2**

For similar best-accuracy reasons, we base our calculation of **ScF** upon bullet data at **1340 fps** airspeed regardless of the actual range to the intended target.

We calculate the potential drag-force **q(T)*S** using the calculated density **ρ** of the ambient atmosphere in **slugs per cubic foot**, and the airspeed **V(T) = 1340 feet per second**:

$$q(T)*S = [(\rho/2)*(1340 \text{ fps})^2]*[(\pi/4)*d^2] \quad (9)$$

This potential drag-force value should be about **1.1 lbf** for a **30-caliber** bullet at this airspeed depending on air density. The potential drag-force at **1340 fps** varies most strongly with the square of the caliber **d(in feet)** of the bullet. The actual drag force **F_D** experienced by the bullet flying at any given airspeed is this potential drag force multiplied by that bullet's coefficient of drag **CD** for the Mach-speed corresponding to that airspeed.

While the drag coefficient **CD** of a rifle bullet *increases* as its airspeed slows in supersonic flight, the actual drag force **F_D** which it experiences continually *decreases* during its supersonic flight. This net reduction is caused by the dynamic pressure **q** decreasing even more rapidly with **V²** as the bullet slows.

The analytic estimate of **β_R(T)** is calculated per **Eq. 6** above with **t = T** and **V(T) = 1340 fps**. With these simplifications **Eq. 6** becomes:

$$\beta_R(T) = -g*\{\exp[-(k\omega + k_v)] - 1\}/[f_2(0)*V(0)*(k\omega + k_v)] \quad (10)$$

If we know the *initial* gyroscopic stability **S_g** of the bullet, we can calculate the initial Stability Ratio **R** of its epicyclic rates **f₁/f₂** from:

$$R = 2*\{S_g + \text{SQRT}[S_g*(S_g - 1)]\} - 1 \quad (11)$$

The *initial* coning rate **f₂(0)** in hertz can then be found from Tri-Cyclic Theory as:

$$f_2(0) = V(0)/[Tw*(I_y/I_x)*(R + 1)] \quad (12)$$

where

Tw = Absolute value of the Twist Rate of barrel in feet per turn.

I_y/I_x = Ratio of transverse to axial second moments of inertia for this bullet. [An algorithm for estimating **I_y/I_x** with reasonable accuracy is explained in the detailed version of this paper. It should also be used for this **I_y/I_x** calculation in the earlier CWAJ paper.]

Substituting back into **Eq. 10**, we have:

$$\beta_R(T) = -g*Tw*(I_y/I_x)*(R + 1)*\{\exp[-(k\omega + k_v)] - 1\}/[V^2(0)*(k\omega + k_v)] \quad (13)$$

where

$V(0)$ = Launch velocity of this particular bullet in feet per second. [$V(0)$ is assumed to exceed **Mach 2.0**]

$k\omega = -0.98604$, as before for any long-range bullet, and

$k_V = \ln[1340/V(0)]$.

This value $\beta_R(T)$ in radians is the estimated Yaw of Repose angle for this bullet when it has slowed to **1340 fps**.

The coefficient of lift value $CL_\beta(T)$ is evaluated based on an estimate of the initial $CL_\beta(0)$ for the Mach-speed of the bullet at launch (here **Mach 2.3289**) from Robert L. McCoy's INTLIFT program for the nose-length effect, but using our own boat-tail effect lift reduction for these long-range bullets.

We multiply McCoy's nose-length estimated CL by the square root of **0.2720/BC7** for each bullet, reasoning that about half of any differing drag for bullets having higher or lower ballistic coefficients **BC7** (Ballistic Coefficient relative to the G7 Reference Projectile) than our example M118LR bullet is due to having a more or less effective boat-tail design.

If a more reliable **BC1** value (BC relative to the G1 Reference Projectile) is available for your rifle bullet, use the square root of **0.5460/BC1** for this CL adjustment for bullet drag.

Very slightly scaling McCoy's published lift curve for the well-studied 30-caliber 168-grain Sierra International bullet to this lift coefficient **2.720** at this **Mach 2.3289** airspeed yields a coefficient of lift $CL_\beta(T)$ of **1.877** at **Mach 1.20** in this standard sea-level ICAO atmosphere.

The bullet's zero-yaw coefficient of drag CD_0 function determines its time-rate of decay in Mach-speed. The supersonic lift-to-drag ratio F_L/F_D for any given angle-of-attack tends to be an invariant aerodynamic characteristic of each basic bullet shape.

Since the coefficients of lift and drag are highly correlated at any given Mach-speed over the rather similarly shaped population of long-range rifle bullets, the same exponential time-decay coefficient:

$$k_L = \ln[CL_\beta(T)/CL_\beta(0)] = -0.3711 \quad (14)$$

can be used in calculating the coefficient of lift $CL_\beta(T)$ for any long-range rifle bullets of interest here.

We propagate this *initial* coefficient of lift $CL_\beta(0)$ estimate forward to its value at time T as:

$$CL_\beta(T) = CL_\beta(0) * \text{EXP}[-0.3711 * (V(0)/2600 \text{ fps})^2 * (1.430 \text{ sec}/T)] \quad (15)$$

The coefficient of lift $CL_\beta(T)$ for any very-low-drag (VLD) or ultra-low-drag (ULD) long-range rifle bullet should be smaller than **1.90** at this airspeed of **1340 fps**. Bullets designed for lower aerodynamic drag by incorporating an effective boat-tail design will also produce

less aerodynamic lift. Conversely, you cannot produce more lift without also increasing drag in aerodynamics.

The exponential propagation function [Eq. 47] estimates a larger fraction of the initial coefficient of lift $CL_{\beta}(0)$ remaining at **1340 fps** airspeed when the time-of-flight **T** *to that airspeed* is increased due to firing a higher-drag bullet, but the initial velocity correction factor $[V(0)/2600 \text{ fps}]^2$ prevents this increase when time-of-flight **T** to **1340 fps** increases simply due to firing that same bullet at a higher muzzle velocity $V(0)$. That is, if the *same bullet* is fired at different muzzle velocities, its estimated coefficient of lift $CL_{\beta}(T)$ at **1340 fps** airspeed should remain the same.

The muzzle velocity $V(0)$ is assumed always to exceed **Mach 2**. This coefficient of lift propagation yields the expected $CL_{\beta}(T) = 1.8769$ at **1340 fps** for the M118LR bullet, and varies by less than **1 percent** over any reasonable launch speeds $V(0)$ for this one bullet type.

The scale factor **ScF** is now calculated from Eq. 8 using the values of the time-functions at time **T** as calculated in Eq. 9, Eq. 13, and Eq. 15 above:

$$ScF = 0.388132*[q(T)*S]*\beta_R(T)*CL_{\beta}(T)/Wt \quad (16)$$

If we might be slightly misestimating the Yaw of Repose angle $\beta_R(T)$ or coefficient of lift $CL_{\beta}(T)$ used here in any systematic ways for these minimal-coning-motion “constant wind” 6-DoF flight simulations, the empirically-determined initial constant factor **0.388132**, from that same PRODAS data, tends to absorb any net difference.

Calculating the Spin Drift at the Target

The spin-drift **SD(tof)** at the *target distance* is then calculated from **Eq. 1** above using the invariant Scale Factor **ScF**, as calculated in **Eq. 16** above, and the total **DROP** from the axis of the bore for the time-of-flight (**tof**) to the target:

$$\mathbf{SD(tof)} = \mathbf{ScF * DROP(tof)} \quad (17)$$

The Spin-Drift **SD** at the target is calculated here in **Eq. 17** in the same *distance* or *angular units* in which the bullet's **DROP** from the bore axis is given. The proper sign depends upon coordinate system conventions and the sense of the bullet's spin.

The bullet **DROP** and time-of-flight (**tof**) to the target are accurately calculated in many existing 3-DoF point-mass trajectory propagators. After all, the accurate calculation of bullet **DROP** at the target distance is the basic figure of merit for these software aids.

The time-of-flight (**tof**) to the target is used in the Litz **SD** estimator and is nice to know even if we do not actually need it in these calculations.

If your particular trajectory propagation program does not directly output “drop from bore axis” data, you can usually “fake” it into doing so by setting your scope height equal zero, setting the angle-of-fire accurately equal to that of the anticipated shot, setting the rifle's “zero range” equal to some minimum distance (ideally zero, or 5 or 10 yards if made necessary by input limit constraints), and specifying that the trajectory calculations go out to the target's known range.

In other words, we want to calculate the **DROP** from the bore axis at the target distance as if we were “bore-sighted” on that long-range target.

The smoothed spin-drift reported by PRODAS at 1000 yards for “constant zero-wind” simulations with this M118LR bullet fired in these conditions is **9.5407 inches**. The spin-drift **SD** estimated via this algorithm for 1000 yards (without the factor of **1.01154** increase in **ScF**) is **9.5635 inches**.

Comparing the two results millisecond-by-millisecond throughout the flight of **1692.3 milliseconds**, yields a mean difference of **0.0043 inches** with a population standard deviation of **0.0207 inches**.

This level of agreement between our analytical estimator for spin-drift for each millisecond and the PRODAS numerical simulation results is rather astonishing. The rounding error for drop and drift data in the PRODAS report format is **0.180 inches** at 1000 yards, and we are not even estimating the horizontal component of the bullet's small coning motion which is included in the PRODAS drift data.

The agreement of this spin-drift **SD** estimator with PRODAS “constant 10 MPH crosswind” runs is almost as good when the Scale Factor **ScF** includes the **1.154 percent** increase as formulated above.

This **1.154-percent-augmented** version of **ScF** in **Eq. 8** should be calculated for firing any *dynamically stable* rifle bullets outdoors.

Example Calculations of Spin Drift

The parameters needed to calculate Yaw of Repose β_R and spin-drift **SD** are calculated for the five example bullets in the spreadsheet shown below. A 3-DoF trajectory program was used to compute the time-of-flight (**tof**) and flight distance to an airspeed of **1340 FPS** and **tof** to a 1000-yard target for the 168-grain SMK bullet, the new 173-grain ULD bullet, and the two Berger 30-caliber bullets tested by Bryan Litz. PRODAS trajectory data was used for the M118LR bullet.

The initial gyroscopic stability factor **Sg** was taken from McCoy for the old 168-grain Sierra International bullet (and applied to the 168-grain SMK), and **Sg** is calculated using McCoy's McGYRO program for the new 173-grain ULD bullet.

PRODAS reports the **Sg**-value for each millisecond of the flight of the M118LR bullet, but we just used their initial value. Bryan Litz gives the initial **Sg** values for the two Berger bullets used in his drift firings.

The ULD bullet is a dual-diameter design with the base of the ogive measuring **0.3002 inches** in diameter (**1.0-calibers** for this bullet design). It has a rear driving-band measuring **0.3082 inches** in diameter (**1.02665 calibers**).

The midpoint (CG) of the rear driving-band is located **1.6 calibers** behind the base of its 3-caliber secant ogive, and the effective width of this driving band is **0.6 calibers**.

Our calculated Yaw of Repose angles β_R for the first three example bullets when each has slowed to an airspeed of **1340 FPS** shows an interesting progression. The estimated Yaw of Repose angles β_R of the three trajectories at the **1340 FPS** airspeed points are **0.471231 milliradians** for the obsolete 168-grain International bullet at **816 yards** downrange, and **0.693417 milliradians** at **888.5 yards** downrange for the M118LR bullet, but just **0.434382 milliradians** for the new 173-grain monolithic brass ULD bullet, and this occurs way beyond the 1000-yard target at **1457 yards** downrange.

The assumed **3200 fps** muzzle velocity of this ULD bullet is based on firing it from a 300 Remington UltraMag cartridge. Each of the other example 30-caliber bullets is assumed to be fired from a much less powerful 7.62 mm NATO or 308 Winchester cartridge.

For comparison purposes the spin-drift **SD** at **1000 yards** is calculated in inches for each of our five example bullets using the **SD** estimator published by Bryan Litz:

$$\text{SD(inches)} = 1.25 * (\text{Sg} + 1.2) * (\text{tof})^{1.83} \quad (18)$$

Our estimates of **SD** at 1000 yards are *smaller* than Bryan's estimates for each of our five example bullets.

Our estimate of **7.0152 inches** of spin-drift **SD** at 1000 yards for the old 168-grain Sierra International bullet from a 12-inch twist barrel is approximately **2.135 inches** less than the **9.15 inches** shown graphically by McCoy in **Figure 9.8** of his MEB, and is **3.005 inches** less than the **10.020 inches** calculated by the Litz estimator for this bullet.

We expected our estimate to be at least **5 percent** (or **0.458 inches**) too small for this *dynamically unstable* bullet. We cannot readily explain the remainder of this difference other than point to the unusually high coning rate $\omega_2(t)$ of this bullet throughout its flight due to its low I_y/I_x ratio.

Our estimate of spin-drift **SD** at 1000 yards for the M118LR bullet **9.7037 inches** matches the **SD** computed by PRODAS (**9.5407 inches**) quite closely (error: **+0.163 inches**, or **+1.708 percent**).

We expected this value to be **1.154 percent** too large because this PRODAS run is for the unrealistic “no wind anywhere” case causing absolute minimum coning motion of this bullet. Thus, the unexplained difference between our **SD** estimate and that computed in PRODAS is **0.554 percent**. The Litz-estimated **SD** of **10.2791 inches** for this M118LR bullet at 1000 yards exceeds the PRODAS value by **0.7156 inches**, or **+7.483 percent**.

Our estimate of **3.1214 inches** of spin-drift **SD** at 1000 yards for the radical new 173-grain monolithic brass ULD bullet design, versus the value of **5.1696 inches** calculated by the Litz estimator for this bullet, indicates the need for our more elaborate **SD** calculation in predicting the long-range flights of current and future ultra-low-lift rifle bullets, even when fired from faster twist-rate barrels.

The Litz spin-drift estimator is closer than our estimator to reported spin-drift values for the old 168-grain Sierra International bullet [McCoy] and for the Berger 175-grain OTM Tactical bullet [from Bryan’s own drift firings].

Our estimator seems closer for the remaining three bullets, especially for the two very-low-drag (and correspondingly very-low-lift) bullets—the brass 173-grain ULD bullet and the Berger 185-grain Long Range Boat-Tail (LR-BT) bullet.

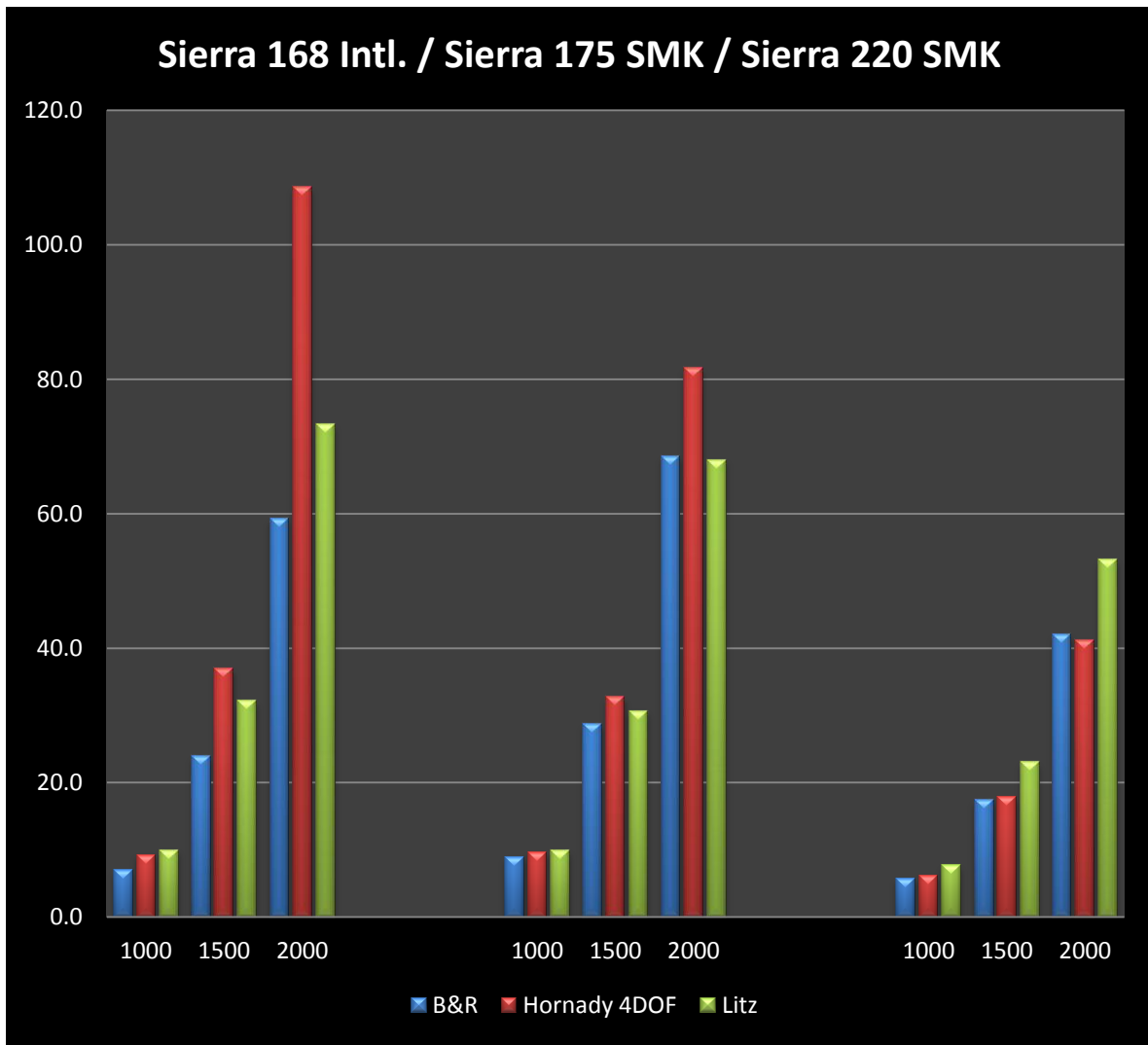
Of course, our predictive agreement with the PRODAS calculations for the M118LR bullet is best of all. We suspect that if 6-DoF simulations could be run for the other four bullets, our estimator would match those results more closely.

We also suspect that Bryan’s drift firing results might not match linear 6-DoF aerodynamic flight simulation results particularly well if they could be computed.

The aerodynamic responses of real rifle bullets are non-linear enough to affect the calculation of these small second-order effects. Real bullets are also subject to other types of aerodynamic jump phenomena in real firings—some of which might be at least partially systematic.

Spin-Drift Example Calculations:				Temp (Degrees-F)	37.00
	Std ICAO Atmosphere:			Rel. Humidity	70.00
Rho (Lbm/Cu. Ft.)	0.0764742	0.0764742	0.0764742	Stat Pressure (InHg)	28.9500
Rho (Slugs/Cu.Ft.)	0.002376894	0.0023768	0.002376	0.002397459	0.00239745
Mach 1.00 = a (FPS)	1116.45	1116.45	1116.45	1093.23	1093.23
30-Caliber Example Bullets:	168-gr International	175.16-gr M118LR	173-gr ULD(SB)	175-gr Berger Tactical	185-gr Berger LR-BT
Bullet Length L (cal)	3.9800	4.0260	5.4368	4.1169	4.3929
Nose Length LN (cal)	2.2600	2.3052	2.8368	2.3701	2.5747
Diameter of Meplat DM (cal)	0.2500	0.2175	0.1000	0.1948	0.2013
Length of Boat-Tail LBT (cal)	0.5100	0.5360	0.7012	0.6331	0.5844
Diameter of Base DB (cal)	0.7645	0.8280	0.8420	0.8409	0.8182
Ratio of Ogive Generating Radii RT/R	0.9000	1.0000	0.5000	0.9000	0.9500
Calc. Tangent Ogive Radius RT (cal)	6.9976	6.9866	9.1666	7.1779	8.4993
Calc. Full Tangent Ogive Length LFT (cal)	2.5976	2.5955	2.9861	2.6321	2.8722
Calc. Full Conical Nose Length LFC (cal)	3.0133	2.9459	3.1520	2.9435	3.2236
Calc. Full Nose Length LFN (cal)	2.6392	2.5955	3.0690	2.6632	2.8897
V0=Launch velocity (FPS):	2800.00	2600.07	3200.00	2660.00	2630.00
Initial Mach-Speed	2.5079	2.3289	2.8662	2.4332	2.4057
Initial B-value	2.3000	2.1032	2.6861	2.2182	2.1880
Ballistic Coef (G1 Ref)	0.4260	0.5460	0.6290	0.5060	0.5530
Ballistic Coef (G7 Ref)	0.2180	0.2720	0.3220	0.2580	0.2830
INTLIFT CL(0)	3.1015	2.7203	2.5551	2.8145	2.6189
Time T to 1340 FPS (sec)	1.2723	1.4300	2.1070	1.3972	1.5030
Range at 1340 FPS Airspeed (yards)	816.00	888.50	1457.00	881.5400	839.1000
Est CL(T) at 1340 FPS:	1.9120	1.8769	1.7447	1.8913	1.8248
Twist Rate (inches/turn, RH)	12.0000	11.5000	8.2500	10.0000	10.0000

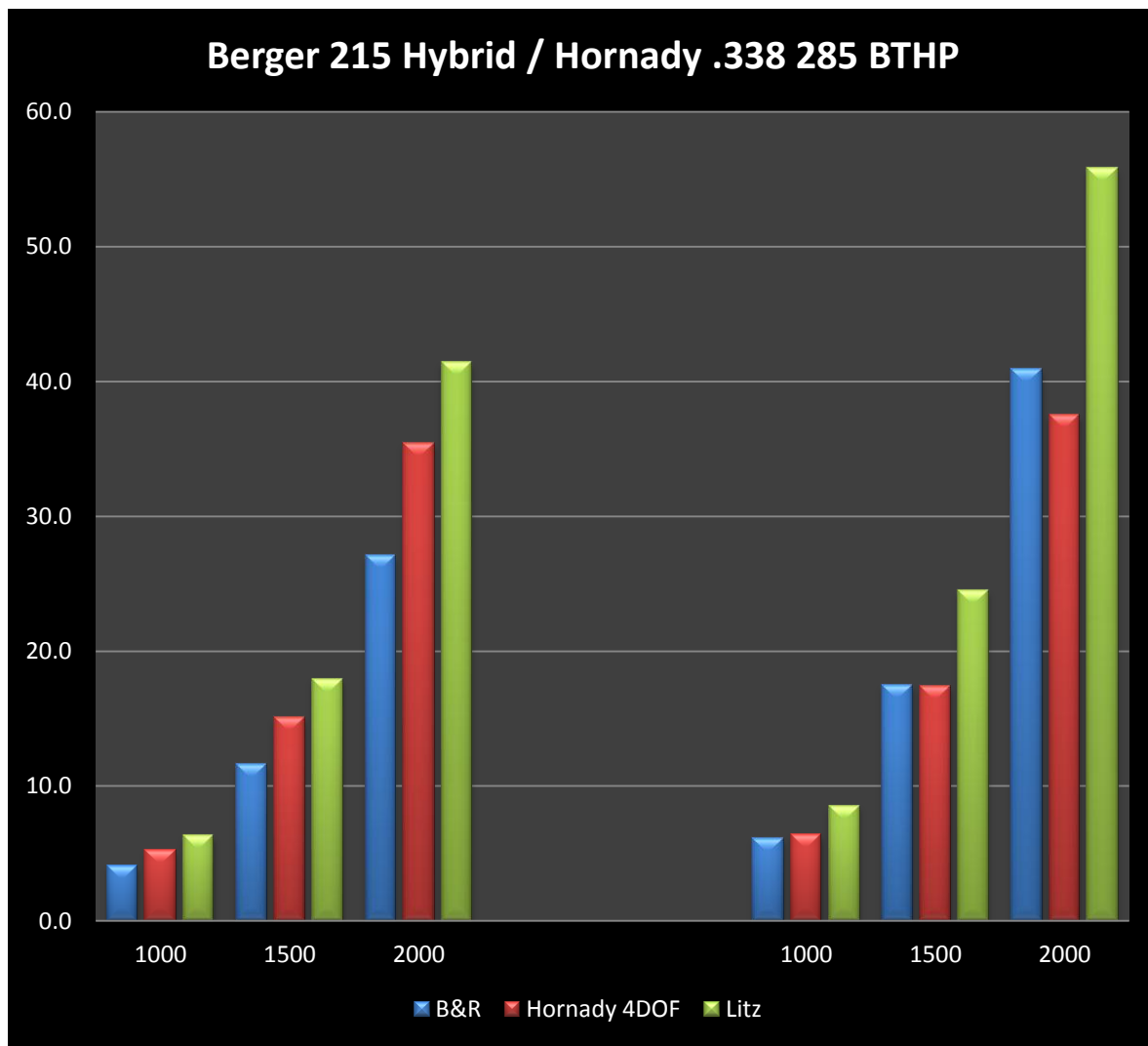
Initial Sg	1.7400	1.9400	1.5940	2.2400	1.9100
Initial Stability Ratio (R)	4.7494	5.5808	4.1341	6.8132	5.4567
Calculated (Iy/Ix) Ratio	7.478685	9.067260	13.46233	9.7336	10.8488
Initial Coning Rate f2 (hz)	65.1188	45.4687	67.3429	41.9719	45.0549
kv=LN(1340/V(0))	-0.736950	-0.662869	-0.870481	-0.685657	-0.674314
komega+kv	-1.722990	-1.648909	-1.85652	-1.671697	-1.660354
Beta-R at time T (mrad)	0.471231	0.693417	0.434382	0.744921	0.696844
Ref. Diam. (1.0 cal. in inches):	0.3080	0.3080	0.3002	0.3080	0.3080
Frontal Area S (square feet)	0.000517403	0.0005174	0.000491	0.0005174	0.00051740
Potential Drag Force at 1340 fps (lbf)	1.1041	1.1041	1.0489	1.1041	1.1041
Bullet Weight (grains)	168.0	175.1600	173.0	175.0	185.0
Bullet Weight (lbf)	0.0240000	0.0250229	0.02471	0.02500	0.0264286
Calculated Scale Factor ScF	0.016088267	0.0222895	0.012484	0.0241500	0.02061980
DROP from Bore Axis at 1000 yds (inches)	436.0450	435.3450	250.0250	428.4970	414.8350
Time of Flight (tof) to 1000 yds (seconds)	1.7300	1.6923	1.2390	1.6870	1.6400
Remaining Velocity at 1000 yds (FPS)	1145.00	1213.99	1836.00	1197.00	1278.00
DROP from Bore Axis at 100 yds (inches)	2.5500	2.6750	2.1400	2.7950	2.8550
Calc. 100-yard SD (inches)	0.0410	0.0596	0.0267	0.0675	0.0589
Calculated 1000-yd Spin-Drift (inches rightward)	7.0152	9.7036	3.1214	10.3482	8.5538
SD from McCoy Figure 9.8	9.1500				
SD from PRODAS runs		9.5407			
SD (inches) from Drift Firings				11.4000	6.7000
Litz Est. Spin-Drift (inches rightward)	10.0203	10.2791	5.1696	11.1967	9.6125
Litz Est. Spin-Drift Minus Our Calc. SD (inches)	3.0051	0.5754	2.0482	0.8485	1.0586



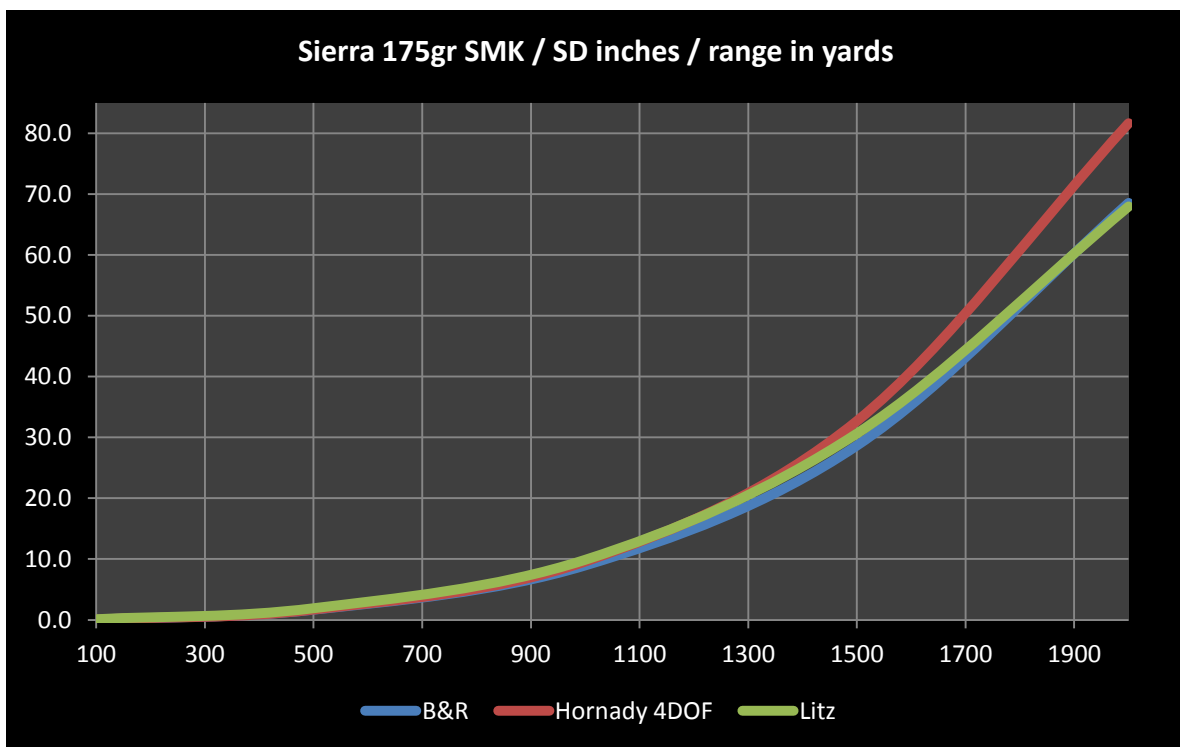
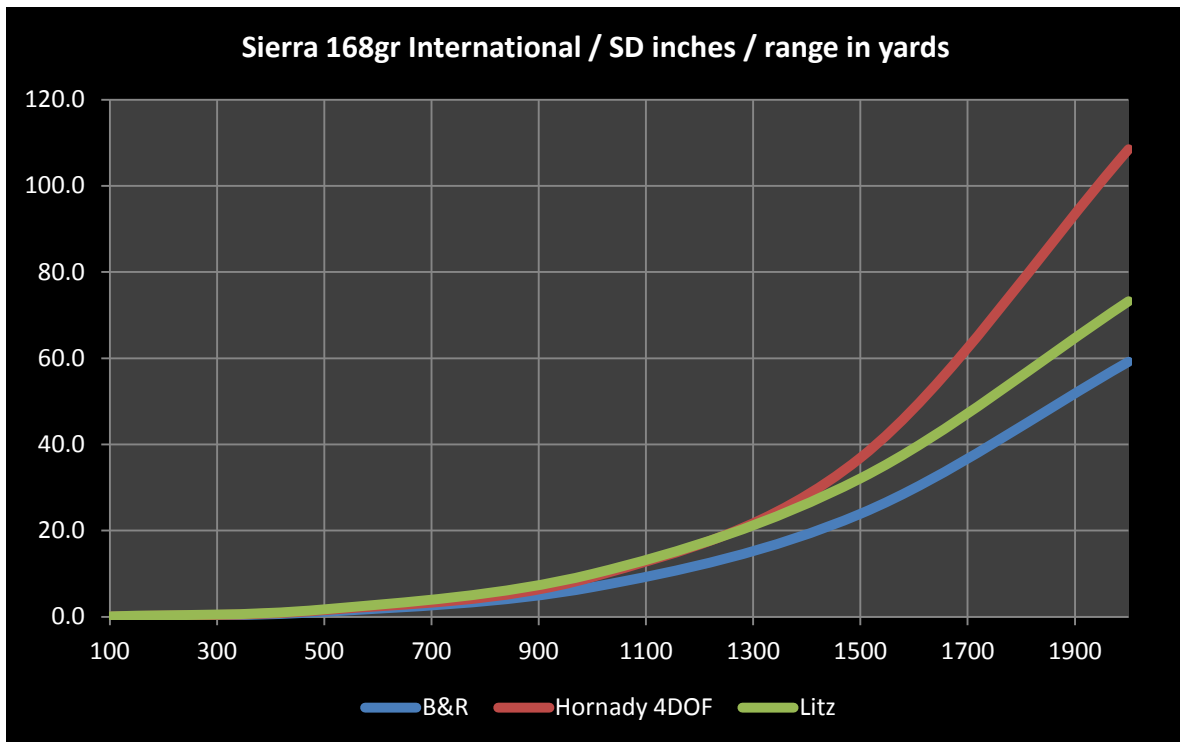
The Spin Drift is expressed in inches, while each bullet is compared with the three different estimators, and grouped at 1000, 1500 and 2000 yards, which are typical ranges for extended Long Range shooting.

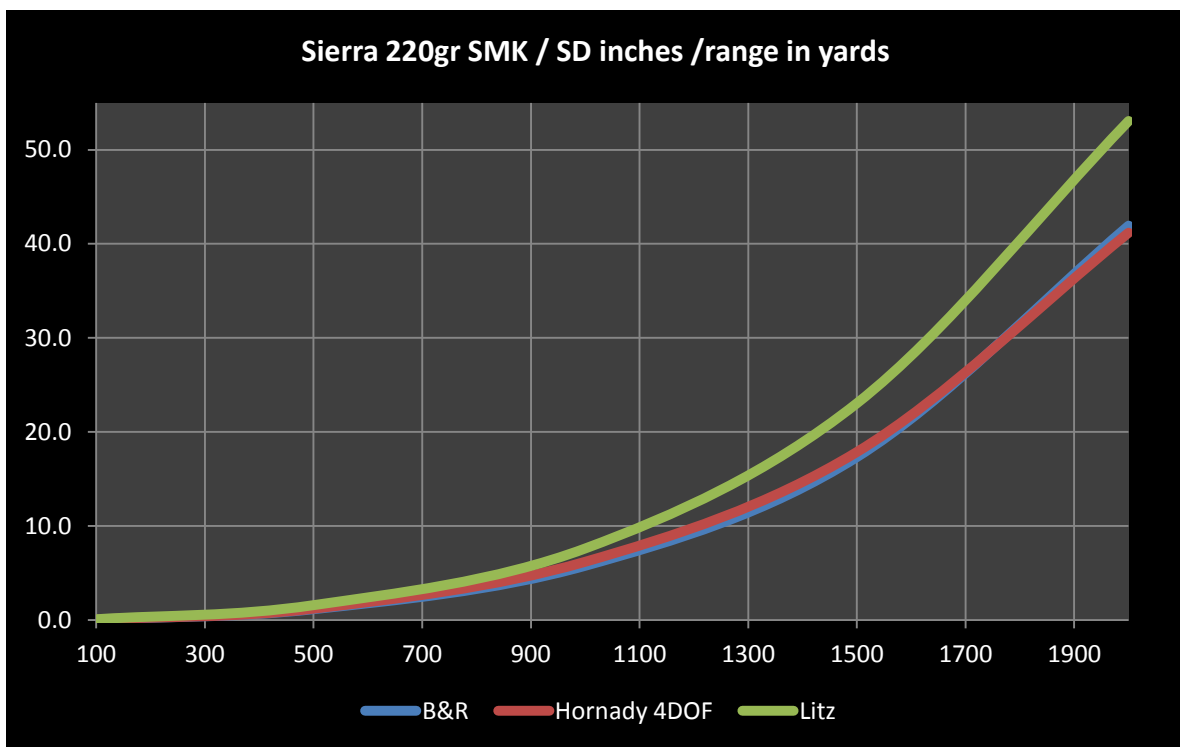
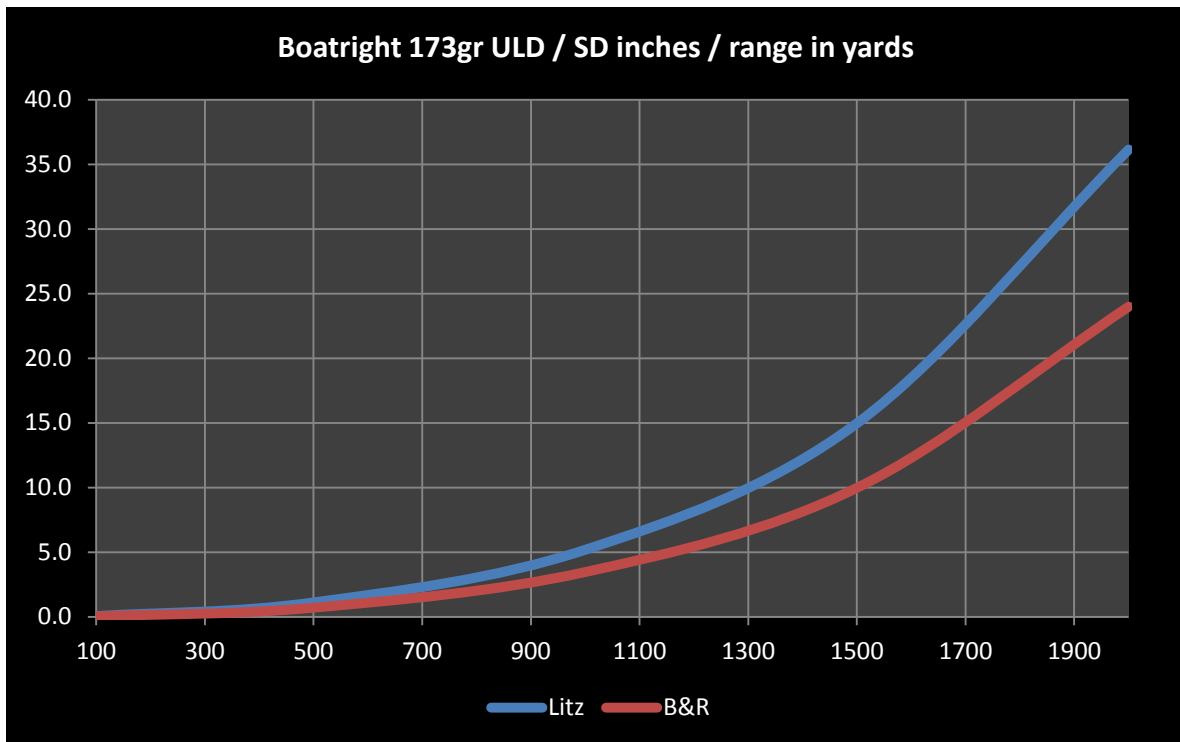
The Litz estimator does fair work, given its simple inputs, but its reliability is dictated by the underlying aerodynamics characteristics of the bullet, which are not accounted for in this simple linear approach.

Consequently, as soon as the bullet does not exhibit certain properties that cannot be encompassed by **Sg** alone, its predictive accuracy is decidedly affected. In general terms, the Litz model tends to over predict **SD** in a significant way.



As can be appreciated, as the range increases, the difference among the estimators becomes larger. The practical side of this is that the correct method is of paramount importance when dealing with Extreme Long Range shooting.





Closing Summary

Taken together, the implications of **Eq. 8** and **Eq. 19** determine the bullet and rifle characteristics which affect the size of the horizontal spin-drift **SD(t)** which will be seen in flat firing at a long-range target.

First, we see from **Eq. 8** that **SD(t)** displacement is always proportional to the bullet's **DROP(t)** in distance units from the projected axis of the bore at firing. This implies that modern lighter-weight “flat shooting” bullets fired at higher muzzle velocities **V(0)** and retaining more velocity farther downrange (higher ballistic coefficient, lower drag bullets) will produce much less spin-drift **SD(t)** at any target distance compared to slower, higher-drag bullets. That is, here **SD(t)** is roughly proportional to time-of-flight **t** to the target distance.

Second, according to **Eq. 19**, the size of the scale factor **ScF**, and thence the size of the spin-drift **SD(t)**, varies directly with the “potential ballistic drag force” $q(t)*S = \rho*V^2(t)*S/2$ in pounds. The ambient atmospheric density ρ varies with shooting conditions. The rifle bullet's retained velocity **V(t)** depends upon its muzzle velocity **V(0)**, its mass **m**, and the integrated drag function **CD_α** of that bullet. The bullet's cross-sectional area $S = \pi*d^2/4$ varies with the square of the bullet's caliber **d**.

Third, the spin-drift **SD(t)** of the bullet is proportional to its yaw-of-repose angle **β_R(t)** throughout its flight:

$$\beta_R(t) = (2\pi*g/t) \int [\omega_2(t)*V(t)]^{-1} dt$$

Both the coning rate **ω₂(t)** and the forward velocity **V(t)** of the bullet always gradually decrease, continually increasing **β_R(t)** throughout the bullet's flight. The coning rate **ω₂(t)** is determined by the bullet's fixed inertial ratio **I_y/I_x** and by the remaining spin-rate **ω(t)** and slowly increasing gyroscopic stability **Sg** of the flying bullet. The forward velocity **V(t)** of the flying bullet depends on its launch velocity **V(0)** and its coefficient of drag profile.

The yaw-of-repose attitude angle **β_R(t)** is *increased* for bullets having larger numerical **I_y/I_x** ratios and higher initial stability **Sg**, but **β_R(t)** is *decreased* by using faster twist-rate barrels and higher muzzle velocities **V(0)** to achieve that higher gyroscopic stability **Sg**.

Fourth, the spin-drift **SD(t)** is directly proportional to the small-yaw coefficient of lift **CL_β(t)** of the bullet. Very-low-drag (VLD) and ultra-low-drag (ULD) bullet designs usually have correspondingly reduced coefficient-of-lift functions at all supersonic airspeeds.

Fifth, and lastly, the spin-drift **SD(t)** of the bullet is inversely proportional to the weight **Wt** (or mass **m**) of that bullet.

All else being equal, bullets made with lower average material densities, such as turned brass bullets, will weigh less and will suffer greater spin-drift.

These five **SD** effects combine multiplicatively in this analysis. Some bullet and rifle design parameters recur in several of these different **SD** effects, and not always working in the same direction.

As modern long-range rifles and their bullets seem to be evolving toward lighter-weight, smaller-caliber, lower-drag bullets fired at higher muzzle velocities, these related incremental variations in design parameters combine algebraically to *reduce the spin-drift SD occurring on long-range targets*.

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