Initial Aeroballistic Yaw and Yaw-Rate

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Several years ago, David Tubb asked me to explain ballistically why rifle performance varied with the barrel's twist direction in the presence of horizontal crosswinds. Several 6-DoF ballistics flight simulation runs showed only perfect left/right symmetry. The reason we could not find the differences was that all the simulation runs used the standard initialization parameters of zero initial yaw and yaw-rate: i.e., assuming the rifle bullets were launched "perfectly" into the local atmosphere.

After formulating the effects of recoil-induced barrel vibrations and calculating them for several target rifle barrels, I can now confidently answer David's question. These barrel vibrations routinely introduce significant vertical plane yaw-rate (tumbling motion) into well-made target bullets as they exit the muzzles of even today's best rifles. While transiting the muzzle blast zone, these bullets then acquire significant vertical plane yaw attitudes (nose up or nose down) before commencing ballistic flight through the undisturbed atmosphere a few feet in front the muzzle.

The rifle bullet's non-zero aeroballistic yaw attitude and yaw-rate at the beginning of the first coning cycle of its free ballistic flight have significant effects upon its entire subsequent trajectory to any target distance. First, non-zero initial vertical plane yaw attitude produces a horizontal "aerodynamic jump" trajectory deflection angles. These horizontal plane trajectory deflection angles typically produce horizontal shot displacements across the target at least comparable in size to the now well-studied vertical displacements resulting from purely horizontal crosswinds directly in front of the firing position. The horizontal shot displacement discussed here would appear to the rifleman as indistinguishable from an unexpected crosswind effect (extra windage) at the target.

This combined windage effect can be flipped from unfavorable (greater windage) to favorable (smaller windage) by pre-selecting the favored rifling twist direction (LH or RH) based upon prevailing left or right crosswind direction at the firing point for any given target rifle design in any given shooting match conditions. However, this extra windage effect can also be ***minimized***, along with its associated ***always unfavorable*** and ***variable*** yaw-drag penalty, by proper tuning of the rifle barrel's vibrations so that the muzzle of the barrel is not vertically accelerating during bullet exit.

The fired bullet establishes its initial coning angle **α** during its first coning cycle of free aeroballistic flight. As discussed here, the initial pointing direction of the bullet's spin-axis is defined by its initial aeroballistic yaw attitude vertically above or below the pointing direction of the forward velocity vector **+V** of the fired bullet. By the end of its first coning cycle, the spin-axis of the bullet is revolving about the "eye of the apparent wind" in the same sense as the bullet rotation imparted by the rifling of the barrel. This revolving motion of the spin-axis direction is termed "coning motion," and its rate of revolution is the gyroscopic precession rate of the spinning bullet. The direction of the "eye of the wind" is determined by the vector difference **V – W**, where **W** is the cross-track component of any horizontal crosswind pertaining at the firing point. Its angular deviation α_{Hor} from the **+V** direction is determined by the ratio of **-W** to **V**.

If the initial yaw and yaw-rate are each **zero**, the initial coning angle **α** is equal to the crosswind offset angle α_{Hor}. When initial vertical yaw and yaw-rate are non-zero, upward or downward, the size **increase** of the initial coning angle **α** depends on the horizontal location of the "eye of the wind" and the sense of the rifling twist. A "favorable" combination results in a ***smaller increase*** in the coning angle **α** when the initial spin-axis vertical pointing offset and motion aligns with the required coning motion of that spin-axis, and vice-versa. This is critically important because the bullet's initial aerodynamic ***yaw-drag penalty*** is proportional to the ***square*** of this initial aerodynamic angle of attack **α** which is identically its initial coning angle **α**. As the flight progresses, the coning angle **α** gradually damps down (exponentially) for dynamically stable rifle bullets until finally the bullet is flying with its maximum ballistic coefficient BC as intended by its designer.

For a rifle bullet, as a spin-stabilized rotationally symmetric projectile, the term "aeroballistic yaw attitude," or just "yaw" as used herein, means the vector sum of its "aircraft type" pitch and yaw attitudes. The "vertical plane aeroballistic yaw" attitudes discussed herein are identical to "aircraft type pitch attitudes." The following analysis is based on data values for the recoil-induced vertical plane vibrational motions at the muzzle at the time of bullet exit for any defined typical target rifle barrel as calculated in an available Excel spreadsheet which requires data input from QuickLOAD© or another interior ballistics calculator for the barrel and cartridge load being used.

1. Estimating the Initial Yaw-Rate

A simple calculation can estimate the initial vertical plane "tip-off" yaw-rate without needing actual bullet mass distribution measurements. If the spreadsheet calculated muzzle lateral acceleration is seen as an impulsive acceleration of the bullet at its point of last contact during exit, that calculated lateral muzzle acceleration **y-double-dot**, upward or downward in the vertical plane, can be converted into a rotational acceleration **α-double-dot** of the fired bullet by dividing **y-double-dot** of the muzzle by the distance $\mathbf{D_{LC}}$ (from the bullet's point of last contact forward to the CG of that bullet) as the **radius** of the bullet's induced yaw rotation:

$$\alpha\text{-double-dot} = \text{y-double-dot}/D_{LC}$$

The axis of any rotational spin or tumbling motion of the free-flying bullet must pass through the CG of that rigid free-flying body. The net lateral force being exerted on the moving bullet by the inside walls of the bore at any time during the bullet's passage is exactly the force needed to ***constrain*** the bullet motion to follow that bore. Consider a bullet ***constrained*** to lying stationary on a tabletop. The total upward force exerted by the table upon the bullet exactly cancels that bullet's weight.

The time duration $\mathbf{\Delta t}$ of this mechanically applied lateral acceleration impulse is given by $\mathbf{D_{LC}/V}$, where **V** is the "muzzle velocity" of the exiting bullet.

$$\Delta t = D_{LC}/V$$

The ***overturning torque impulse*** resulting from the constraining force applied against the outside of the departing bullet by the bore inside the muzzle crown increases ***linearly*** from zero at CG passage to its maximum value at the point of last contact, a distance $\mathbf{D_{LC}}$ behind that CG location. Thus, we calculate the angular acceleration effect of the overturning impulse as ***one half*** its full-strength lateral acceleration value multiplied by the bullet's exiting time interval **Δt**. An estimate of the initial yaw-rate **α-dot** can then be calculated as

α-dot = 0.5*(α-double-dot*Δt)

or **α-dot = 0.5*(y-double-dot/V)**

The yaw-rate **α-dot** so calculated is given in **radians per second**, or equivalently in **milliradians per millisecond** (**mrad/msec**). Note that the distance $\mathbf{D_{LC}}$ from point of last contact to CG for the bullet being fired "divides out" and thus, does not need to be measured. Greater lateral muzzle acceleration **y-double-dot** forcing produces larger initial yaw-rate, while greater bullet exit speed **V** produces less initial yaw-rate.

Spreadsheet calculations of vertical lateral acceleration rates at the muzzle indicate that **y-double-dot** values of **1,000 to 1,500 meters per second squared** might be common for untuned or mistuned rifles. These high rates of acceleration are attributable to the high ***flexural rigidity*** of the steel rifle barrel. Muzzle exit speed **V** of a rifle bullet typically runs about **750 to 1,000 meters per second**. Thus, initial tumbling yaw-rates should run up to about **0.50 to 1.0 mrad/msec**, either vertically upward or vertically downward at the nose of the bullet. A milliradian (mrad) is often called a "mil" by riflemen. The initial yaw-rate of the bullet's nose is positive (upward) when the lateral force on its rearward point of last contact is downward, and vice versa.

2. Initial Vertical Yaw Attitude

A supersonic bullet fired from a rifle must first traverse the turbulent muzzle blast flight zone before commencing aeroballistic free flight through the ambient atmosphere where its airspeed might be about **Mach 2.5**. After travelling about **2** to **4 meters** from the muzzle, the supersonic rifle bullet penetrates the front-most muzzle blast shockwave and enters the undisturbed ambient atmosphere. At this point, the blast shockwave is progressing at only **Mach 1.0** in the ambient atmosphere. The bullet's typical time-of-flight through this muzzle blast flight regime is from about **2.0 msec** up to perhaps about **5.0 msec**.

If the estimated typical vertical yaw-rate of **0.5 to 1.0 mrad/msec** pertains throughout the **2.0 to 5.0 msec** duration of transiting the muzzle blast zone, the bullet would develop an initial vertical yaw component of **1.0 to 5.0 mrad** in the **same vertical direction** as the initial yaw-rate by the time free aeroballistic flight can begin. At least during the early part of its transit of the muzzle blast zone, the bullet is being pushed along and bypassed by a much higher speed jet of hot powder gasses from the muzzle behind it. Any non-zero initial yaw and yaw-rate can only be ***amplified*** in magnitude by the "reverse aerodynamics" overturning moments acting on the yawed rifle bullet during this passage. How much amplification is anyone's guess, but likely varies a lot. Studies of the gas flows exiting rifle barrels show they are quite unstable and vary greatly from shot to shot. Gyroscopic reaction of the spin-stabilized bullet to this reversed overturning moment will not be significant due to its brief duration. The resulting variations in initial aeroballistic yaw attitude and yaw-rate are detrimental to rifle and bullet performance and are best avoided by tuning the vibrating barrel for near-zero lateral acceleration during bullet exit.

There is another systematic type of initial vertical plane aeroballistic yaw attitude imparted by vertical plane muzzle speed **y-dot** during bullet exit:

$$\alpha_{y\text{-dot}} = -(y\text{-dot})/V$$

where **V** again represents the exit velocity of the bullet. The vertical muzzle speed **y-dot** is calculated from barrel vibrations in the spreadsheet. Since **y-dot** at the muzzle of a vibrating barrel is typically less than about **100 mm/sec**, this initial yaw attitude $\alpha_{y\text{-dot}} \leq 0.1$ **mrad** is usually small. This small vertical plane yaw $\alpha_{y\text{-dot}}$ is algebraically added into the initial vertical plane yaw attitude discussed above.

3. Initial Horizontal Yaw Attitude

The bullet's initial horizontal aeroballistic yaw attitude α_{Hor} due to encountering a full value **10 MPH (4.47 m/sec)** horizontal crosswind **W** either from **3:00** or **9:00 o'clock** at the firing point would be about **4.5** to **6.0 mrad**, depending upon the muzzle speed **V** of the fired bullet.

$$\alpha_{Hor} = -W/V$$

The yaw attitude at the beginning of free aerodynamic flight would then be the vector sum of the vertical and horizontal rectangular initial yaw attitude components. This aerodynamic attitude is relative to the horizontally offset "eye of the wind" which defines the "apparent wind" as seen by the flying bullet. A properly constructed and fired target rifle is assumed to produce no barrel vibrations in the horizontal plane while being fired.

From Coning Theory, each rectangular component of the initial yaw attitude produces its own "aerodynamic jump" trajectory deflection angle, and each leads the "roll orientation" of its initial aeroballistic yaw angle component by 90-degrees in the sense of the barrel's rifling twist. That is, with a right-hand twist barrel, a rightward initial horizontal yaw (due to a purely horizontal crosswind from left-to-right at the firing point) produces a downward jump deflection of the entire remaining trajectory out to any target distance. Similarly, a vertically upward initial yaw attitude produces a rightward aerodynamic jump which would appear as added crosswind deflection (windage) when firing through that L-to-R crosswind. These effects reverse both with initial vertical yaw direction (up/down based on barrel tuning) and with the selected rifling twist sense (LH/RH). When the initial vertical direction yaw and yaw-rate are each essentially zero, the initial coning angle of attack $\boldsymbol{\alpha}$ is just $\boldsymbol{\alpha_{Hor}}$ as determined above from the crosswind **W** and bullet speed **V**.

4. Other Aeroballistics Effects of Initial Yaw-Rate

Other than creating a non-zero yaw attitude in the vertical plane during passage of the rifle bullet through the muzzle blast zone, the aeroballistic effects of this non-zero vertical ***yaw-rate*** are not well understood. As when poking a spinning top, the initial yaw-rate produces a gyroscopic "fast mode" nutation (or "nodding") bullet motion superimposed on the gyroscopic "slow mode" precession (or coning motion) of the spin-stabilized bullet. Fast mode nutations do ***not*** typically produce trajectory “aerodynamic jump” deflections and usually damp out rapidly for most rifle bullets.

However, non-zero initial yaw-rates ***might well*** produce significantly different aerodynamic yaw-drag effects on the entire remaining trajectory of the bullet during the beginning of coning motion as aerodynamic flight commences, especially in the presence of crosswind at the firing point. This would occur because the initial coning angle **α** is always larger than $\boldsymbol{\alpha}_{Hor}$ as it would be with **zero** initial yaw-rate. This coning angle **α** is identically the aerodynamic “angle of attack” for the flying bullet, and the aerodynamic drag acting to retard that bullet varies with the square of that angle of attack **α**. Airspeed lost early in flight due to unnecessary yaw-drag cannot be recovered once the Ballistic Coefficient (BC) improves due to dynamic damping of the coning angle **α** far downrange.

I speculate that the initial size of the bullet’s coning angle of attack **α** will be increased somewhat by favorable (and significantly increased by unfavorable) initial yaw-rates and directions when firing through any crosswind **W** at the firing point. These coning angle and aerodynamic jump effects occur during the first half of the first coning cycle of aeroballistic free-flight, and would typically produce no further trajectory deflection disturbances after about the first **50 feet** of bullet travel, as long as the crosswind **W** remains constant. With access to a 6-DoF aeroballistic flight simulator, a series of a few dozen runs should sufficiently illuminate this situation.

5. Summary

Let us term the “ideal baseline” type of long-range rifle shooting to be launching each bullet with as near to **zero** aeroballistic yaw and yaw-rate and with as nearly the same bullet speed **V** as we can achieve. These bullets should survive passage through the turbulent muzzle blast zone with minimal yaw disturbance. They will suffer coning angle of attack $\boldsymbol{\alpha}_{Hor}$ induced vertical direction aeroballistic jump and initial yaw-drag penalty caused by the crosswind **W** pertaining at the firing point. They will also suffer what I term “deDion type” windage displacement across the target based on wind conditions from muzzle to target and total flight delay time due to air-drag retardation.

A target rifleman can select from two approaches to improve the bullet launching performance of his rifle and ammunition toward that ideal. One, he can bring a matched pair of rifles, or interchangeable rifle barrels, differing only in the twist direction of their rifling (right-hand and left-hand twist directions) to each shooting event. By selecting the favored twist direction for each event based upon the prevailing crosswind direction at his firing point, he can expect to encounter ***less*** windage displacement on target than with baseline deDion windage. However, his shots will fly with ***somewhat lower*** initial BC than ideal, and with more variance in BC encountered from shot to shot due to firing a yawing bullet through the muzzle blast zone. Use of a properly designed muzzle brake might help reduce this initial BC variance to acceptable levels by controlled venting of the propellant gasses.

Second, the rifleman might tune his match rifle's barrel vibrations for minimum lateral acceleration at the muzzle during bullet exit. Barrel vibration analysis can be used backed by shooting tests searching for maximum consistent initial BC measurements with the selected bullet. This "super-tuning" would minimize the bullet's initial yaw and yaw-rate, together with their variances acquired during passage through the muzzle blast zone. The ideal baseline windage, aerodynamic jump, and initial BC performance should be consistently delivered by each fired bullet with minimum variance. Because maximizing initial BC and minimizing the variations in BC observed from shot to shot are probably even more important in long-range rifle shooting than minimizing error in windage estimation due to crosswinds between muzzle and target, this super-tuning approach is recommended.

Studies using my barrel vibration calculating spreadsheet indicate that shorter length barrels are easier to super-tune than longer barrels. The spreadsheet indicates it is possible to super-tune a 24-inch barrel for simultaneous minima in lateral acceleration and in lateral velocity at the muzzle at the calculated time of bullet exit. This tuning basically involves playing Mode 2 and Mode 3 barrel vibrations against each other, which is only possible when neither is strongly dominant (as with short rifle barrels).

If the ballistic advantages of greater length barrels are needed (as for ELR shooting, for example), the long barrel can be ***barrel block*** mounted to reduce its vibrating length to that exposed beyond the barrel block. The added mass of the barrel block is symmetrical about the bore axis and should help minimize the CG offset of the rifle system in its firing position. The bedded barrel block transfers the force of recoil to the rifle stock.

Other rifle design techniques which help control recoil induced barrel vibrations are:

- Cantilever mounting the scope to the barrel block,
- Free-floating the action from the stock and scope, and
- Use of a rigid, but very low mass rifle stock design.

If the CG of the assembled rifle system can be made to fall very nearly on the bore axis, perhaps the entire problem of vertical plane barrel vibrations can be avoided, just as left/right symmetry currently avoids recoil induced horizontal plane barrel vibrations.