

# Interior Ballistics with Copper Bullets

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## 1.0 Introduction

Monolithic copper bullets have not always behaved during their interior ballistics phase quite as well as we have come to expect with conventional jacketed, lead-cored rifle bullets. Bullet designers, barrel makers, gunsmiths, and riflemen need to understand these differences and limitations to avoid disappointing performance when switching to copper bullets.

We will use the well accepted behavior characteristics in the rifle barrel of premium quality, soft-jacketed, soft-lead-cored, match-type rifle bullets as our benchmarks for interior ballistics performance which we strive to equal or better in firing our monolithic copper rifle bullets.

We will compare the interior ballistics performance characteristics of a new 338-caliber 246-grain monolithic copper Ultra-Low-Drag (ULD) bullet of my own patented design against those of the more conventional 338-caliber 250-grain Sierra MatchKing bullet. The promising new copper bullet is currently well into development and testing.

Our firing tests are done using a single-shot, bolt-action match rifle fitted with various premium quality custom barrels, each using different rifling patterns and rifling twist-rates, but all chambered in 338 Lapua Magnum using a custom chambering reamer.

In particular we shall compare (1) the barrel obturation characteristics, (2) the shot-start pressures required, (3) the bullet/barrel friction characteristics, and (4) the bullet spin-up effects of these two example bullets.

The interior ballistics performance formulations developed herein should be generally applicable to rifle bullets of different materials, construction, and design and to those of other rifle calibers as well.

Our first analysis will be formulating the amount of internal barrel expansion to expect at peak base-pressure behind each 338 bullet. This unexpectedly large **0.0013-inch** internal barrel expansion is at the root of many problems encountered in firing hard monolithic copper bullets.

## 2.0 Physical Properties of Materials Used

The materials being studied here are limited to

- ***Essentially Pure Lead***, used as the core material in many benchrest and match-type rifle bullets
- ***Gilding Metal Copper Alloy***, used as the jacket material in these bullets
- ***Essentially Pure Copper***, from which our monolithic bullets are made by CNC machining
- ***Rifle Barrel Steel***, from which our various match-grade rifle barrels are made.

Most military and hunting-style rifle bullets are made with harder core and jacket materials than are used when match accuracy and consistency are the main goals. Pure lead is the softest and most dense of the continuum of lead alloys commonly used in making lead bullets and the cores of jacketed bullets. Benchrest rifle bullet cores and many other match bullet cores are usually made of essentially pure lead, and we use it here both as the ideal limiting case and as the one core material for which its material properties are most widely known. Benchrest bullet makers prefer the softest and most dense lead for precisely pressure-forming (“squirting”) the cores which will subsequently be pressure-seated into the jackets through their open fronts. Bullet cores made from lead alloyed with a small amount of antimony, such as our example Sierra MatchKing bullet here, are only slightly less dense and slightly tougher and stronger than those made from essentially pure lead.

We are mainly concerned here with the friction characteristics of the gilding metal jacket alloy composed of approximately 95-percent copper and 5-percent zinc. The thin walls of the bullet jacket (**0.032-inch** for our example bullet) cannot contribute much strength in preventing the jacketed bullet from distorting as readily as does its lead core. The jackets may be left in

their work-hardened state as formed when making tough military and hunting bullets, but they are usually annealed in some step before the final bullet-pointing operation when making benchrest or match-type rifle bullets.

The copper material from which our prototype ULD bullets are made by CNC turning is half-hard, cold-rolled rod stock of UNS C147 99.6-percent Copper with a trace amount (up to 0.4 percent) of sulfur added for improved machinability. This half-hard copper state was formerly designated H04, but is now called H02. The hardness of the copper bullet material is critically important in interior ballistics. Annealed copper (H00) is not only much weaker, both in yield strength and in shear strength, but is also notoriously “grabby” in friction.

We will use average or typical physical properties of the many types of carbon-steel alloys, which include several stainless-steel alloys, from which premium quality custom barrel blanks can be made using button-rifling or cut-rifling techniques.

The physical properties of these four materials are tabulated in the table below. Many of these specifications are converted from metric pressure units **MPa** into pounds per square inch (**psi**). One Mega-Pascal (**MPa**) is equivalent to **145.04 psi**. We will also freely use the abbreviation **ksi** which is commonly used in ballistics work to stand for **kilo-pounds per square inch**, or **thousands of psi**.

<u>Materials Properties:</u>	<u>Lead</u>	<u>Gilding Met.</u>	<u>Copper</u>	<u>Barrel Steel</u>	<u>Units</u>	
<u>Density</u>	11.34	8.86	8.92	7.85	grams/cc	
<u>Weight</u>	2867.80	2240.70	2255.80	1985.20	grains/cu in	
<u>Yield Strength (S)</u>	1,740	10,000	40,000	130,000	psi	
<u>Modulus of Elasticity (E)</u>	2,030,000	17,000,000	16,900,000	29,000,000	psi	
<u>Poisson's Ratio (<math>\mu</math>)</u>	0.44	0.33	0.33	0.30	none	
<u>Coeff of Friction (Cf)</u>	1.40	0.22	0.36	0.42	Sliding on dry steel	

### **3.0 Physics of Interior Ballistics**

An introductory level understanding of the physics of elasticity and strength of materials is necessary to understand how rifle bullets and barrels behave in interior ballistics. Even though the pressures involved are fairly high by everyday standards and the barrel dwell times of rifle bullets are only one to two milliseconds, static and quick-static loading analyses are adequate for our purposes here in which we are primarily concerned with radial and tangential effects as opposed to axial-direction effects. We shall use the British Engineering units of feet, inches, pounds, and grains commonly used by American riflemen instead of the SI (international metric) units used professionally.

Much of the numerical data we use comes from interior ballistics simulation runs of the QuickLOAD© commercially available software program. We use QL frequently in load development and recommend it to riflemen. As a principal “reality check,” we compare QL calculations of expected muzzle speeds with measurements made using barrel-attached Magneto-Speed and 5-yard downrange Oehler 35P chronographs.

We will develop the elementary concepts and relationships of material strength, stresses and strains, distributed force and pressure, and sliding friction in each section as we need them.

### **4.0 Barrel ID Expansion with Internal Pressure**

We look first at the internal expansion of our rifle barrels in firing because it is larger than most riflemen would believe and because it is critically important in our analyses of copper bullet performance in interior ballistics.

We assume that the peak chamber pressure in firing our 338 Lapua Magnum rifle cartridges is its CIP-specified Maximum Average Pressure of **60 ksi** (thousands of pounds per square inch). We use the interior ballistics program, QuickLOAD©, to estimate the *peak base-pressure P* behind our example bullets at **51 ksi** or **85 percent** of that peak chamber pressure. Either of our example bullets will have travelled only **3.2 inches** down the barrel at the time when the gas pressure acting on the base of the bullet reaches its peak value. The same peak base-pressure pertains when firing either jacketed, lead-cored bullets or copper bullets of the same weight and having similar shot-start pressures.

Our example Heavy Varmint profile 338-caliber match barrels have an outside diameter ( $2*r_o$ ) of **1.24 inches** at that bullet location, as very nearly does another **1.26-inch** cylindrical 338-caliber barrel fitted for our test rifle. We shall use the nominal groove diameter of **0.3380 inches** as the inside diameter ( $2*r_i$ ) of each barrel.

The internal pressure  $P_i$  within the rifle barrel at this point of peak base-pressure  $P$  must also include the significant peak radial contact pressure  $\sigma_{rcp}$  exerted by the bullet upon the adjacent walls of internal barrel steel.

As will be shown later, the peak radial contact pressure  $\sigma_{rcp}$  is

$$\sigma_{rcp} = 24.6 \text{ ksi}$$

for our copper bullet at this *same time and location* in the barrel. So, the total internal pressure  $P_i$  being contained within the rifle barrel at peak base-pressure on the bullet is the *sum* of this internal gas pressure  $P$  and the internal mechanical contact pressure  $\sigma_{rcp}$  of the bullet:

$$P_i = P + \sigma_{rcp} = 51.0 + 24.6 = 75.6 \text{ ksi}$$

In normal operation, the peak internal pressure  $P_i$  should not exceed **100 ksi** in good steel rifle barrels for safety reasons.

As will also be shown later, the peak radial contact pressure  $\sigma_{rcp}$  of the bullet at this point when firing the jacketed bullet is essentially the same at **22.4 ksi**, so the barrel expansion will be quite similar when it is being fired.

Any rifle barrel qualifies mechanically as a thick-walled cylindrical pressure vessel which is not normally constrained in its axial-direction (lengthening or contracting) with increased internal pressure  $P_i$ . Fortunately, we have Lamé's Equations for calculating analytically the two-dimensional (radial and tangential) stresses and the amount of radial expansion displacement  $U(r)$  for any element within the steel walls at radius  $r$  ( $r_i \leq r \leq r_o$ ) from the axis of the rifle barrel as functions of the pressure  $P_i$  being applied internally. These equations hold for any rifle barrels made of isotropic materials, (such as any of our barrel-making steels) which have not been pre-stressed, and which are not stressed in operation beyond their proportional elastic limits.

The operative form of Lamé's Equation for finding the radial displacement  $U(r)$  for an element of the barrel material at any radial location  $r$  within a pressurized thick-walled cylindrical pressure vessels is:

$$U(r) = (P_i * r / E) * [(1 - \mu) + (1 + \mu) * (r_o / r)^2] / [(r_o / r_i)^2 - 1]$$

where

$U(r)$  = Radial expansion in inches at any radius  $r$  from axis  
( $r_i \leq r \leq r_o$ )

$P_i$  = Internal pressure in psi = 75,600 psi here

$E$  = Young's Modulus of Elasticity = 29,000,000 psi  
(average for barrel steels)

$r_o$  = Outside radius of cylinder = 0.620 inches here

$r_i$  = Inside radius of cylinder = 0.169 inches here

$\mu$  = Poisson's Ratio = 0.30 for most barrel steels.

In particular, by setting  $r = r_i$ , the calculated value  $U(r_i)$  becomes the internal *radial* expansion of the inside walls of the pressurized barrel:

$$U(r_i = 0.1690 \text{ inches}) = \underline{0.000643 \text{ inches.}}$$

And the internal *diameter* (ID) expansion is 1.287 thousandths of an inch.

We might also calculate the outside diameter (OD) expansion of this rifle barrel over this same part of the bore by setting  $r = r_o$ :

$$U(r_o = 0.620 \text{ inches}) = 0.000260 \text{ inches.}$$

As expected, the outside diametral expansion over this point is *much less* at just **0.520 thousandths of an inch**. This shows how external strain gauges can allow indirect laboratory measurement of base-pressures behind the bullet up and down a test barrel.

Had our 338-caliber barrel been of lighter profile with a **0.75-inch** OD at this point ahead of the chamber swell, for example, the internal diameter expansion at this point of peak base-pressure would have been **1.595 thousandths of an inch**.

Note that these ID and OD elastic expansions with internal pressure  $P_i$  are not dependent on the quality, heat treatment, and strength ratings of the barrel steel. These internal barrel expansions are quite accurately

calculated provided the barrel material has no *implanted* stress caused by being stressed beyond its elastic limit.

Button-rifling is a barrel rifling process which *necessarily* over-stresses a thin-walled cylinder of the internal barrel steel surrounding the bore and leaves behind a residue of implanted tangential (hoop) stresses which actually *do* reduce subsequent bore expansions with internal pressures. Without conducting a detailed analysis, we estimate that button-rifled barrels will expand internally by about **2/3** as much as would a similar profile and caliber stress-free cut-rifled barrel at similar internal pressures. That is, the implanted hoop stresses will *cancel* about the first **1/3** of any pressures subsequently applied internally. For this reason, a similar over-stressing operation, called “autofrettage,” is performed hydraulically or mechanically during the manufacturing of artillery tubes.

We should also note that this internal expansion of the rifle barrel has the same basic timing as the chamber pressure curve. The peaking impulse of this chamber pressure looks like the first half of a sinusoidal pressure wave of about **1200 hertz**. This is the fundamental driving frequency for many types of barrel vibrations.

This rapid internal expansion of the barrel generates acoustic shear pressure waves which travel up and down the length of the rifle barrel, propagating axially at from **10,230 fps** (stainless steel) to **10,630 fps** (AISI 4340). Longitudinal acoustic pressure waves travel almost twice this fast in steel. In shear waves, the particle motion is perpendicular to the direction of propagation. These acoustic shear waves reflect back and forth from the impedance changes at the muzzle and breach ends of the rifle barrel and sum algebraically at any point along the barrel. Best rifle accuracy is obtained when the multiply-reflected shear waves cause minimal internal barrel expansion at the muzzle just at the time of bullet exit. This appears to be the physical basis of empirical ***Optimum Barrel Time*** calculations.

### **5.0 Jacketed Lead-Core Bullets as Obturation Benchmark**

Conventional jacketed, lead-cored rifle bullets typically obturate conventionally rifled barrels quite well, at least well enough for military and big game hunting rifle applications. We will discuss barrel obturation by our example match-type jacketed rifle bullet made with a softer gilding metal jacket and using a soft core material of essentially pure lead. We will

compare the **radial contact pressures**  $\sigma_{rcp}$  of the bullet exterior gas-sealing surfaces against the inside groove surfaces of the rifle barrel when firing our example jacketed match bullet versus our new copper bullet.

From years of experience in precision shooting of match-type jacketed, lead-cored rifle bullets, we know that the default shot start pressure of **3625 psi** (as used in QL) produces muzzle speed calculations which agree well with our chronograph measurements. QL recommends using a shot start pressure of **6500 psi** with any copper bullet. For any given load, this higher shot start pressure causes much higher peak chamber pressures and produces just slightly higher muzzle speeds. We hope our new bullet design made of half-hard copper produces shot-start pressures no higher than those of the jacketed match bullet.

We will also compare the bullet friction characteristics for these two bullets travelling through the rifle barrel as typified by their frictions at peak base-pressure **P**. Any significant differences in friction would relate to expected barrel wear and barrel fouling and cleaning differences.

We will use the boat-tailed, 250-grain, 338-caliber Sierra MatchKing bullet as our idea example match-type bullet for comparison of internal ballistics properties. It can be fired quite similarly to our 246-grain copper bullet for these comparisons.

This lead-cored bullet has a shank length of **0.507 inches** at its **0.3384-inch** outside diameter. At the rifling-engraved depth of **4.2 thousandths of an inch**, the engraved length of the bullet is **0.603 inches**. The gilding metal jacket measured **0.032-inch** in thickness on a sectioned sample bullet, leaving a lead core diameter of **0.274 inches** in the bullet shank.

## **6.0 Barrel Obturation by Rifle Bullets**

The sealing, or obturation, of the hot gasses produced during combustion of gunpowder propellants is a significant concern in interior ballistics. Leakage of some of the hot gasses past the projectile in the barrel causes minor damage to the barrel and to the projectile. More severe gas blow-by causes more severe damages and is responsible for much of the observed variation in muzzle velocities from one shot to the next. Highest speeds are measured for shots where barrel obturation was best, and reduced speeds are produced when propellant gasses blow by the bullet at peak base-

pressure and thereafter. Early gas leakage damages the sealing surfaces of the bullet and promotes continued gas leakage during the remainder of that bullet's trip through the rifle barrel.

Direct evidence of damage to a monolithic copper or copper-alloy jacketed bullet is shown by the uniform deposition of metallic copper inside the bore of a rifle barrel toward its muzzle. Elemental copper is vaporized by the hot powder gasses leaking past or blowing by the bullet at peak base-pressure. This hot copper vapor rapidly expands and cools ahead of the bullet and precipitates out on the cold steel surfaces of the barrel toward the muzzle. This copper is easily removed in routine barrel cleaning, as distinct from "copper fouling" which is the shearing of copper particles from the bullet due to rough interior surfaces in the throat or bore of the barrel.

Additional direct evidence of gas blow-by can be seen in high-speed video of a rifle being fired. A dense cloud of smoke often obscures the emergence of the bullet itself from the muzzle.

One useful approach to improving barrel obturation during firing is to select barrels rifled in a way which improves this sealing of the powder gasses with any rifle bullet fired. Better obturation is supported by selecting rifling patterns where (1) the sides of the rifling lands are sloped significantly outward toward their bases, where (2) the bottom inside corners at the groove edges are significantly radiused, and where (3) the top edges of the rifling lands are slightly radiused at both outside corners. Boots Obermeyer's **5R** and Gary Schneider's **P5** rifling patterns come to mind as ideal rifling designs for promoting better bullet obturation. While the author has no experience using it, polygonal rifling should also promote better barrel obturation.

Proper barrel obturation by the rifle bullet is a more complex problem than most riflemen realize. This is largely because the inside diameter (ID) of the 338-caliber rifle barrel has expanded by almost **0.0013-inch** at the instant when the peak base-pressure **P** is driving a 338-caliber bullet down the barrel. This is also the very time when gas obturation is most critical and most difficult for the bullet being fired to accomplish. We will show how monolithic copper rifle bullets can be designed to deal with this internal expansion just as well as do conventional jacketed, lead-cored match bullets.

## **6.1 Barrel Obturation with Jacketed Bullets**

The soft lead core of a conventional jacketed match bullet readily deforms or “slugs up” plastically so that the bullet OD easily matches even the ***pressure-expanded ID of the steel rifle barrel*** at a peak base-pressure **P** of **51 ksi** driving the bullet forward. This behavior of jacketed, lead-cored rifle bullets explains why riflemen used to shooting jacketed, lead-cored bullets are not generally aware of the significant amount of internal barrel expansion which has been happening all along. This permanent, plastic, bullet diameter enlargement is accompanied by a commensurate plastic foreshortening of the lead core within its jacket.

The initial OD of the jacketed bullet and the unstressed ID of the barrel are not particularly critical dimensions for barrel obturation. In fact, lead-cored bullets of the next larger, or smaller, caliber in the same parent cartridge case have sometimes been fired accidentally, without damage to the rifle.

With a value of only **2,030,000 psi** for Young’s Modulus of Elasticity for pure lead (**E<sub>L</sub>**), this lead core material has virtually no elastic “memory” of its pre-stressed shape. This is why lead is called a “dead metal.” You deform it, and it just sits there waiting to be deformed again.

Neglecting the weak, thin gilding metal jacket, the radial contact pressure **σ<sub>rcp</sub>** of the jacketed bullet against the steel walls of the barrel at peak base-pressure **P** is

$$\sigma_{rcp} = \mu_L * \sigma_a = 0.44 * 51 \text{ ksi} = 22.4 \text{ ksi.}$$

The total internal pressure **P<sub>i</sub>** within the barrel at this point is then

$$P_i = P + \mu_L * P = (1 + \mu_L) * P = 73.4 \text{ ksi.}$$

This radial stress **σ<sub>rcp</sub> = 22.4 ksi** is well above the yield strength (**S<sub>L</sub>**) for the pure lead core of just **1740 psi**. This pure lead core material is acting almost as an incompressible liquid, at least as far as the transfer of pressures is concerned.

As the base-pressure **P(t)** subsequently drops and the rifle barrel begins returning elastically to its unstressed ID, the radial contact pressure **σ<sub>rcp</sub>(t)** at points farther down the barrel becomes

$$\sigma_{rcp}(t) = S_L + \mu_L * P(t) = 1.74 + 0.44 * P(t) \text{ ksi}$$

This peak  $\sigma_{rcp}$  value of **22.4 ksi** for a lead-cored rifle bullet gives us a good indication of how much radial contact pressure  $\sigma_{rcp}$  is required for any rifle bullet to seal **51 ksi** of gas pressure effectively within any typical rifle barrel.

If  $\sigma_{rcp}(t)$  were always to equal or exceed the base-pressure  $P(t)$ , one might then say that “perfect mechanical obturation” had been achieved between the two smooth surfaces. These copper-alloy jacketed, lead-cored rifle bullets have about 125 years of development and testing behind them.

## **6.2 Barrel Obturation with Copper Bullets**

Monolithic copper bullets can be designed so that they achieve **effective obturation** of the hot powder gasses throughout the interior ballistics portion of their firing process. A 338-caliber copper ULD bullet of my own patented design is used here as an illustrative example.

Some monolithic bullet designs utilize a sequence of narrow, over-diameter gas sealing rings which are designed always to be plastically compressed during rifling engravement. A similar approach is used in artillery shell designs. Plastic deformation in compression always includes a *maximum elastic compression* at the elastic limit for the material of the sealing rings. A downside of using multiple small gas sealing rings for monolithic copper rifle bullets is the necessarily higher aeroballistic drag induced by the multitude of secondary shock waves which these sealing rings invariably throw off during supersonic and transonic flight. Another disadvantage of using narrow sealing rings for barrel obturation is that their elastic “working length” in providing contact pressure is only the compressed height of the sealing rings themselves, typically just a few thousandths of an inch, as opposed to the full radius of an engraved bullet shank or of a much wider driving/sealing band. These are among the reasons we selected a rear driving/sealing band design for our monolithic copper ULD bullet designs.

Understanding basic physical concepts can increase our understanding of what happens to a monolithic copper rifle bullet in the interior ballistics phase of rifle firing. Inside the rifle barrel, the copper bullet does not act entirely as a “rigid body” as it does for all practical purposes in exterior ballistics. The copper bullet material is subjected to stresses large enough to cause significant elastic and some plastic deformations in its shape. We need to understand how these bullet distortions might affect the ability of

the copper bullet to seal, or obturate, the hot powder gasses most effectively and thereby minimize any damaging gas blow-by during firing.

Let us start by looking at the rifling engraved bullet at the moment of peak chamber pressure (**60.0 ksi**, in this example) which occurs when the bullet has travelled just a few inches down the rifle barrel (**3.2 inches** here) from its initial position. According to the interior ballistics program *QuickLOAD*® the peak base-pressure **P** accelerating our example 246-grain copper 338-caliber bullet is

$$\mathbf{P} = 51.0 \text{ ksi}$$

which is **85 percent** of that peak chamber pressure at this point in this 338 Lapua Magnum example.

The base-pressure **P(t)** gradually becomes a smaller fraction (**82.5 percent** at the muzzle) of the instantaneous chamber pressure as the bullet speed increases and as it travels down the somewhat gas-flow restrictive 338-caliber barrel. Base-pressure **P(t)** is only **10.8 ksi** behind the bullet at the muzzle of our example **28-inch** barrel.

The peak base-pressure **P** exerts a distributed force **F** on the bullet accelerating its entire mass forward. The area **A<sub>B</sub>** over which this force **F** is distributed can best be thought of as the cross-sectional area of the rearmost barrel-obturing aperture, an imaginary plane within the rearmost full diameter portion of the rifle bullet. This plane area **A<sub>B</sub>** is also ideally equal to the cross-sectional area of the rifled bore of the obturated barrel.

The bullet material in front of this obturing aperture is being shoved forward by the distributed force **F(t)**, while any afterbody (boat-tail) material of the monolithic bullet behind this plane is actually being dragged along via its mechanical attachment to the shank of the bullet.

At any instant, we can reason that, hydrostatically,

$$\mathbf{F(t)} = \mathbf{P(t)} * \mathbf{A_B}$$

That distributed force **F(t)** produces a peak axial-direction stress  $\sigma_a$  on the copper bullet material within this obturing aperture which is given, at the instant of *peak* base-pressure, by

$$\sigma_a = \mathbf{F(t)/A_B} = \mathbf{P} = 51.0 \text{ ksi}$$

Note that this peak axial stress  $\sigma_a$  depends only on the base-pressure  $P$  and is completely independent of the caliber of the bullet.

If that axial stress  $\sigma_a$  does not exceed the elastic limit  $S = 40,000$  psi for this “half hard” copper material, it will produce an axial, elastic compressive strain ratio  $\epsilon_a$  in this thin disc of bullet material given, in accordance with Hooke’s Law, as

$$\epsilon_a = \sigma_a/E = P/E = 0.003018$$

where  $E = 16,900,000$  psi is Young’s Modulus of Elasticity for this copper bullet material. While the bullet material within the obturating aperture is compression stressed somewhat beyond its elastic limit  $S$  in this example, it is confined within the steel barrel and has nowhere else to go.

We also know that this compressive axial stress  $\sigma_a$  would, if not constrained by the steel walls of the barrel, produce a radial elastic strain ratio  $\epsilon_r$  on this same bullet material within the obturation aperture given by

$$\epsilon_r = \mu*\sigma_a/E = \mu*P/E = 0.000996$$

where  $\mu = 0.33$  is Poisson’s Ratio of lateral shrinkage to elongation during tensile and compressive laboratory testing of this copper material.

At this strain ratio  $\epsilon_r$ , The potential radial increase  $\Delta r$  in the size of the unconstrained copper bullet would be

$$\Delta r = r*\epsilon_r = 0.1693*0.000996 = 0.000169 \text{ inches.}$$

We can see why this is so if we consider in isolation the thin disc of copper material within the obturating aperture. Unstressed, this thin disc has a radius  $r$  and a thickness of  $h$ . Under axial compressive stress  $\sigma_a$  within its elastic range, the thickness of the disc is reduced to  $h - \Delta h$ , and its radius would increase to  $r + \Delta r$ , being unconstrained here.

If the unstressed *Volume* of this disc,  $A_B*h$ , were to remain *constant* under this axial elastic stress  $\sigma_a$ , we could say

$$\pi*r^2 *h = \pi*(r + \Delta r)^2 *(h - \Delta h)$$

Dividing through by  $\pi*r^2 *(h - \Delta h)$ , and neglecting higher-order differential terms, this expression simplifies to

$$(\Delta r)/r = 0.5*( \Delta h)/h$$

or, in terms of radial and axial elastic strain ratios

$$\epsilon_r = 0.5 \cdot \epsilon_a$$

This “constant volume” condition actually holds only for an “incompressible” liquid such as water in low-pressure hydrostatics. It is almost statically true for a “perfectly elastic” material such as soft gum rubber which has a value approaching **0.5** for Poisson’s Ratio ( $\mu$ ). Here, we must replace the value **0.5** with **0.33**, Poisson’s Ratio ( $\mu$ ) for copper:

$$\epsilon_r = \mu \cdot \epsilon_a = \mu \cdot \sigma_a / E = \mu \cdot P / E$$

Multiplying through by Young’s Modulus **E**, we find that the radial stress  $\sigma_{rbp}$  caused by the peak base-pressure **P** acting axially is

$$\sigma_{rbp} = \mu \cdot \sigma_a = \mu \cdot P = 16.8 \text{ ksi}$$

We now see why Poisson’s Ratio  $\mu$  can never exceed **0.5** for any solid material which retains internally some portion of any stress applied to it. The ability of a metal object to retain stress internally allows it to retain a “memory” of its pre-stressed shape.

Now, let us consider what happens when this unstressed copper aperture sealing disc *exactly fits* the interior of the rifle barrel at this point where maximum base-pressure **P** is to be applied to it. That is, let us assume for the moment that it has **zero** unloaded radial contact pressure  $\sigma_{r0}$ . Let us also assume for the moment here that the much stronger steel walls of the rifle barrel do not move outward with these interior pressure stresses.

When the base-pressure **P** is applied, the radial contact pressure  $\sigma_{rcp}$  is

$$\sigma_{rcp} = \sigma_{r0} + \sigma_{rbp} = 0 + \mu \cdot P = 16.8 \text{ ksi}$$

Perhaps we could achieve more perfect barrel obturation by starting instead with a non-zero static contact pressure  $\sigma_{r0}$ . We could have radially compressed an over-diameter copper bullet in the throat of the barrel by an amount  $\Delta r$  so that its static contact pressure  $\sigma_{r0}$  is

$$\sigma_{r0} = E \cdot (\Delta r) / r \leq S$$

where the upper limit **S = 40,000 psi** (here) is the rated yield strength of our half-hard copper bullet material, and **r** is the radius of our unstressed copper bullet’s sealing surface.

One should design the maximum outside diameter (OD) of the copper bullet shank or its driving/sealing band so that, within bullet manufacturing tolerances, the bullet OD will always be at least as large as the maximum groove inside diameter (ID) for standard specification rifle barrels of each caliber.

With a bullet diameter production tolerance of **+/- 0.0002 inch** (or even less) for these CNC-turned half-hard copper bullets, we specify a rear driving/sealing band OD **0.0006-inch** larger than the **nominal groove ID** for standard barrels of that caliber. Then, when specifying the chambering reamer design, we specify a ball seat inside diameter **0.0008-inch** larger than this nominal groove ID to minimize gas blowby before the bullet enters the throat of the rifle barrel.

Match grade barrel production groove ID tolerance can be specified to fall between the **nominal groove ID** for that caliber and the **nominal groove ID+0.0002 inch**, with less than **0.0001-inch** variation, end to end. Thus, any production bullet within specifications will freely enter the ball seat and **statically** seal the throat of the fitted production barrel blank during firing.

For our 338-caliber copper bullet,  $\Delta r$  is nominally **0.0003 inch**, and  $r$  is **0.1693 inch**, so

$$\epsilon_{r0} = (\Delta r)/r = (0.0003/0.1693) = 0.001772 < \epsilon_{Max}$$

The “half hard” copper from which these bullets are manufactured has a guaranteed minimum yield strength rating **S** of **40,000 psi** and a Modulus of Elasticity **E** of **16,900,000 psi**. Thus, its maximum possible elastic strain ratio  $\epsilon_{Max}$  is given by

$$\epsilon_{Max} = S/E = 40,000/16,900,000 = 0.002367$$

The selected nominal radial compression  $\Delta r = 0.0003$  inches produces **75 percent** of the *maximum possible* radial stress pre-load, and

$$\sigma_{r0} = E*(\Delta r)/r = 30.0 \text{ ksi}$$

If the production and wear tolerances stack so that the radial compression  $\Delta r$  has its minimum value of **0.00020 inches**,

$$\sigma_{r0} = 20.0 \text{ ksi.}$$

If the production and wear tolerances stack so that  $\Delta r$  has its maximum value of **0.00040 inches**,

$$\sigma_{r0} = 40.0 \text{ ksi}$$

The total radial contact pressure now looks pretty good at a nominal

$$\sigma_{rcp} = \sigma_{r0} + \sigma_{rbp} = 30,000 + 16,800 = 46.8 \text{ ksi}$$

which is almost the base-pressure  $P = 51.0 \text{ ksi}$ , and would almost produce “perfect mechanical obturation” as mentioned earlier.

But we have to go back and account for the steel walls of the barrel expanding by  $\Delta r = -0.000643 \text{ inches}$ . The change  $\sigma_{rexp}$  in copper radial bearing stress due to internal expansion of the steel barrel would be given by

$$\sigma_{rexp} = E(\Delta r)/r = 16,900,000*(-.000643/0.1690) = -64.3 \text{ ksi}$$

However, we limit  $\sigma_{rexp}$  to **-40.0 ksi** here, because more than the maximum implantable stress cannot be removed by relaxing the barrel constraint on bullet OD.

And the total radial bearing stress  $\sigma_{rcp}$  of the copper bullet inside the expanded barrel is now given by

$$\sigma_{rcp} = \sigma_{r0} + \sigma_{rbp} + \sigma_{rexp} = 46,800 - 40,000 = 6.8 \text{ ksi}$$

This is now only about **30-percent** of the gas sealing pressure **22.4 ksi** of the lead-cored bullet studied earlier and will not likely seal the gasses effectively. However, there is another way to increase this radial bearing stress for monolithic copper bullets by a designed amount, and that is by drilling their bases axially to port the base-pressure inside the gas sealing portion of those bullets.

### **6.3 Base-Pressure Ducting by Base-Drilling of Copper Bullets**

Large potential (unconstrained) diameter increases or corresponding (constrained) increases in radial contact pressures are available by porting, or ducting, the peak base-pressure  $P$  forward within the body of the monolithic copper bullet by axially base-drilling those bullets. To be effective in improving bullet obturation of the barrel, the base-drilling must pass completely through the boat-tail and at least mostly through under the rear driving/sealing band of this bullet design. This idea of ducting the

base-pressure into a hollowed-out the base of a rifle bullet goes back in history at least to 1849 and the hollow-base musket ball developed by French Army Captain Claude-Étienne **Minié** for the muzzle-loading rifled muskets of that era.

The photograph below illustrates the difference in gas sealing efficiency for prototype monolithic copper 338-caliber ULD bullets fired at 3000 fps into a swimming pool and recovered for study. These prototype bullets have **0.3302-inch** diameter bore-riding shanks and **0.3382-inch** rear driving/sealing bands. The angle of incidence into the water was about 45 degrees, which explains the neatly curled copper ogives. The test barrel was a 10-inch twist Krieger barrel with a conventional six-narrow-land cut-rifling pattern. The bore diameter of the barrel was **0.3302 inches**, and the groove diameter was **0.3380 inches**.

The top bullet is an un-fired example. The second bullet was fired into the pool, but it had not been base-drilled. The bottom two fired bullets had been base-drilled to the depths indicated by the black annotation marks using a **0.166-inch** diameter drill.

Note the very poor gas sealing of the second (un-drilled) bullet. The engraved groove-diameter copper bullet contacts only the middle third of the bottoms of the barrel grooves. Compare that to the two base-drilled bullets also shown. Their engraved rear driving/sealing bands look similar to the shanks of recovered soft-jacketed bullets. Also, notice the evidence of elastic bullet expansion just to the shoulder depth of the base drilling indicated by the black annotation marks.

Porting the base-pressure inside the bullets caused none of the plastic deformations seen in these fired bullets other than the slightly more noticeable rifling marks seen on the bore-riding shanks of the deep-drilled bullets. We have never seen any evidence of unburned powder packing into these hollow bases in many recovered test bullets which had been base-drilled with different drill diameters and to different drill depths.



With proper base-drilling, the rearmost aperture-sealing portion of the rear driving/sealing band becomes a thick-walled cylindrical pressure vessel for which Lamé's Equations can be used to calculate internal stresses and radial displacement as functions of the base-pressure  $P$  ducted internally.

The operative form of Lamé's Equation for radial displacement is:

$$U(r) = (P \cdot r / E) \cdot [(1 - \mu) + (1 + \mu) \cdot (r_o / r)^2] / [(r_o / r_i)^2 - 1]$$

where

$U(r)$  = Radial expansion at radius  $r$  from axis of cylinder  
( $r_i \leq r \leq r_o$ )

$P$  = Internal pressure in psi = 51,000 psi here

$E$  = Young's Modulus of Elasticity = 16,900,000 psi (Cu)

$r_o$  = Outside radius of cylinder = 0.1693 inches here

$r_i$  = Inside radius of cylinder = 0.0830 inches here

$\mu$  = Poisson's Ratio = 0.33 for Cu.

Here, we are calculating the maximum temporary elastic radial expansion  $U(r)$  of the rear driving band of our 338-caliber copper ULD bullet as a function of the radial position  $r$  from the cylinder axis for  $r_i \leq r \leq r_o$ .

In particular, we want to find the potential unconstrained radial expansion at the outside diameter of the copper bullet. Setting  $r = r_o$  for this special case, Lamé's Equation above reduces to:

$$U(r_o) = (2*P*r_o)/\{E*[(r_o/r_i)^2 - 1]\}$$

Using our numerical values for this copper 338-caliber ULD bullet, we have:

$$U(0.1693) = (2*51,000*0.1693)/(16,900,000*3.1606)$$

$$U(0.1693) = 0.0003233 \text{ inches}$$

Thus, the outside *diameter* of the rear driving band could temporarily increase by a calculated **0.000647 inches** when a hydrostatic pressure of **51.0 ksi** is applied to the inside of the obturating surface of the hollow-base copper bullet. This radial expansion is *purely elastic* because  $U(r_o) = 0.0003233 \text{ inches}$  is less than the maximum elastic radial expansion of

$$\Delta r_{\text{Max}} = 0.1693*S/E = 0.0004007 \text{ inches}$$

for these half-hard copper bullets. When this internal pressure drops to **zero psi** (actually less than ambient air pressure) during subsequent aeroballistic flight, the bullet returns to its (engraved) original shape.

This temporary diameter increase significantly improves the obturation of the monolithic copper ULD bullet ***exactly when it is most needed***. Fired test bullets recovered from the waters of a swimming pool show ***essentially perfect obturation*** of these base-drilled bullets forward to the shoulder depth of the internal drilling. Except for the small penalty in ballistic coefficient (BC) caused by the reduction in bullet weight, no other ill effects in bullet behavior are caused by base-drilling. Base-drilling simultaneously improves the gyroscopic stability of the monolithic copper bullets significantly.

Now, if we constrain this potential bullet OD expansion due to base-drilling to exert instead a radial pressure  $\sigma_{\text{rbd}}$  against the inside of the barrel walls, with  $\Delta r = 0.0003233 \text{ inches}$ , the elastic expansion when using this **0.166-inch** base-drill diameter, this radial contact pressure would be given by

$$\sigma_{\text{rbd}} = E*(\Delta r)/r = 16,900,000*(.0003233/0.1693) = 32.3 \text{ ksi}$$

If we reduce the base-drill diameter ( $2*r_i$ ) from **0.166-inch** to **0.125-inch**, the expression  $[(r_o/r_i)^2 - 1]$  in the denominator of Lamé's Equation increases from **3.1606** to **6.3376**, just about cutting the potential elastic diametral expansion in half, from **0.647 thousandths** to **0.3225 thousandths of an inch**.

Correspondingly, with  $\Delta r = 0.0001612$  inches with the smaller base-drill,

$$\sigma_{rbd} = E*(\Delta r)/r = 16.1 \text{ ksi}$$

The timing of this base-pressure-ducting bullet expansion is the same as that of the inertial-force-driven bullet expansion, a reduced amplitude and slightly delayed version of the chamber pressure curve.

With the **0.166-inch** drill size, the total copper bullet radial contact pressure  $\sigma_{rcp}$  would be

$$\sigma_{rcp} = 6,800 + 32,300 = 39.1 \text{ ksi}$$

and with only a **0.125-inch** drill size, it is

$$\sigma_{rcp} = 6,800 + 16,100 = \underline{22.9 \text{ ksi}}$$

Thus, the **0.125-inch** base drill size looks about right for this 338-caliber, and perhaps similar sizes of copper ULD bullet designs. The bullet weight penalty of **10.2 grains** for this base-drilling is an acceptable trade with this **256-grain** (solid-weight) 338-caliber monolithic copper ULD bullet design for the resulting improvement in gas sealing. This **0.125-inch** base drilling produces just about the same total  $\sigma_{rcp}$  value as when firing a comparable soft-jacketed, soft lead-cored match bullet (**22.4 ksi**). However, we must scale down this base-drill diameter for smaller-caliber copper bullets of this same ULD design to minimize weight reduction.

#### **6.4 Summary of Barrel Obturation**

The total combined radial bearing pressure  $\sigma_{rcp}$  of the copper bullet sealing against the inside steel surfaces of the barrel at peak base-pressure and beyond is the instantaneous **sum**, according to the Principle of Superposition, of these five different independently analyzed effects:

$$\sigma_{rcp} = \sigma_{r0} + \sigma_{rbp} + \sigma_{rexp} + \sigma_{rbd} + \sigma_{rcf}$$

These five different contact pressure effects are:

(1) The contact pressure due to initial compression of the **0.0006-inch** over-diameter copper bullet

$$\sigma_{r0} = (0.0006/0.3380)*E = 30.0 \text{ ksi}$$

[This contact pressure component is modeled here as being constant after rifling engravement.]

(2) The dynamic radial stress due to axial stress  $\sigma_a$  at **P = 51.0 ksi**

$$\sigma_{rbp} = \mu*P = 0.33*51000 = 16.8 \text{ ksi}$$

[This is just the bullet expanding due to inertial acceleration.]

(3) The loss in copper bearing stress due to barrel expansion by  **$\Delta r = -0.000643$  inches**, here limited at **-S**,

$$\sigma_{rexp} = -40.0 \text{ ksi}$$

[For a pre-stressed button-rifled barrel, we estimate  **$\Delta r \approx -0.00042$  inches** with a corresponding contact pressure loss of **40.0 ksi**, still, at this barrel expansion.]

(4) The radial stress due to ducting of the base-pressure **P** internally by axially drilling the bullet base with a **0.125-inch** diameter drill

$$\sigma_{rbd} = 16.1 \text{ ksi}$$

[The radial stresses  $\sigma_{rbp}$ ,  $\sigma_{rexp}$ , and  $\sigma_{rbd}$  each have the timing of the base-pressure **P(t)** acting upon the bullet; that is, a reduced and delayed version of the chamber pressure curve.]

(5) The radial stress caused by centrifugal force as the bullet spins up

$$\sigma_{rcf} = 13.7 \text{ ksi}$$

at the full rotation rate of the copper bullet near the muzzle of a very fast twist (**6-inches/turn**) barrel. At the point of peak base-pressure being evaluated here, the bullet is moving at just **1272 fps** and has a bearing pressure  $\sigma_{rcf}$  due to centrifugal force of

$$\sigma_{rcf} = 1.7 \text{ ksi}$$

[The maximum  $\sigma_{rcf}$  is **10.0 ksi** at the muzzle for the comparable jacketed, lead-cored bullet at its maximum rotation rate. Due to its necessarily slower rifling twist-rate, this centrifugal force is negligible at the time of peak base pressure **P** for the jacketed bullet.]

Thus, the total radial contact pressure  $\sigma_{rcp}$  at the critical time of peak base-pressure **P** sums to

$$\sigma_{rcp} = \sigma_{r0} + \sigma_{rbp} + \sigma_{rexp} + \sigma_{rbd} + \sigma_{rcf}$$

$$\sigma_{rcp} = 30.0 + 16.8 - 40.0 + 16.1 + 1.7 = \underline{\underline{24.6 \text{ ksi}}}$$

This contact pressure **10 percent** more than that of a soft lead-core match bullet (**22.4 ksi**) fired under the same conditions. Thus, the base-drilled copper bullet should obturate the rifle barrel at least as well as does that soft jacketed bullet.

Analyzed in terms of bullet expansion into the pressure-expanded rifling grooves of the barrel, the lead-cored bullet has exceeded its elastic limit of **1.7 ksi** by **20.7 ksi** at peak base pressure **P**. The elastic limit of radial expansion for the half-hard copper 338-caliber bullet is **0.0004-inch** in addition to re-expansion of the **0.0003-inch** initial compression of its oversize driving band. As this sum **0.0007 inches** exceeds the radial internal expansion of the steel barrel, this copper bullet does not distort *plastically* other than its rifling engravement. Each bullet will distort either *plastically* or *elastically* so as to fill, and seal, the **0.000643-inch** radially expanded rifling grooves with equal effectiveness.

While study of the elasticity and strength of materials is far more complex than this elementary discussion would indicate, this analysis is sufficient for our purposes here.

## **7.0 Barrel Friction with Copper and Jacketed, Lead-Cored Bullets**

We will compare the friction of the two bullets at the time of peak base-pressure, which is also when peak barrel internal expansion occurs. This comparison will also indicate any differences in friction characteristics to be expected elsewhere in the barrel.

From the values of radial contact pressure  $\sigma_{rcp}$  calculated earlier, we will formulate the total normal contact force **F** distributed over the contact area **A<sub>c</sub>** for each bullet. We can ignore the rifling lands for now and make

“smooth bore” friction calculations for the nominal groove ID of the barrel  $D_g$ . We then find the force-of-friction  $F_f$  resisting bullet motion by multiplying this normal force  $F$  times the Coefficient of Sliding Friction  $C_f$  for the outer material of each bullet sliding smoothly over clean, dry barrel steel.

The actual bullet to barrel coefficient of dynamic friction  $C_f$  experienced during any given shot is far too complicated to ever deal with analytically. The interior barrel surfaces are seldom the “clean, dry steel” mentioned, especially after the first shot, and even the type of barrel steel selected and its micro-finish might be important variables here. Friction modifiers are also used at times.

Yet, somehow rifles can often fire bullets which exit the barrel uniformly enough to produce repeatable accuracy at long ranges. Our purpose here is only to develop some basic physical relationships to allow comparative friction evaluations of our two different bullets, not an attempt to provide hard numerical performance predictions.

For comparison purposes, we will calculate a “friction equivalent” partial base-pressure  $P_f$  for each bullet.

From the bore obturation studies of these two bullets, we found their radial contact pressures  $\sigma_{rcp}$  at **51.0 ksi** peak base-pressure to be:

$$\sigma_{rcp} = 22.4 \text{ ksi} \quad (\text{Jacketed bullet})$$

$$\sigma_{rcp} = 24.6 \text{ ksi} \quad (\text{Copper bullet})$$

The contact area  $A_c$  of each bullet can be formulated as:

$$A_c = \pi * D_g * L_c$$

where  $D_g = \text{Nominal Groove Diameter} = 0.3380 \text{ inches}$

$L_c = \text{Axial Length of Contact Patch}$

$$L_c = 0.507 \text{ inches} \quad (\text{Jacketed bullet})$$

$$L_c = 0.233 \text{ inches} \quad (\text{Copper bullet}).$$

Then  $A_c = 0.5384 \text{ sq. in.} \quad (\text{Jacketed bullet})$

$$A_c = 0.2474 \text{ sq. in.} \quad (\text{Copper bullet})$$

The total normal contact force  $F_C$  distributed over the contact area  $A_C$  is then

$$F_C = \sigma_{rcp} * A_C$$

and the aggregate axial force-of-friction  $F_f$  opposing bullet motion is then

$$F_f = C_f * F_C = C_f * \sigma_{rcp} * A_C$$

Now, the partial base-pressure  $P_f$  equivalent to this axial force-of-friction  $F_f$  spread over the cross-sectional area of the bore  $A_B$  is

$$A_B = (\pi/4) * D_g^2 = 0.08973 \text{ sq. in.}$$

$$P_f = F_f / A_B = C_f * \sigma_{rcp} * (A_C / A_B)$$

$$P_f = C_f * (4 * L_C / D_g) * \sigma_{rcp}$$

Using the values for our two bullets:

$$P_f = 0.22 * (4 * 0.507 / 0.338) * 22.4 \text{ ksi}$$

$$= 29.6 \text{ ksi} \quad \text{(Jacketed bullet)}$$

and

$$P_f = 0.36 * (4 * 0.233 / 0.338) * 24.6 \text{ ksi}$$

$$= 23.2 \text{ ksi} \quad \text{(Copper bullet)}$$

While these two “friction” pressures are neither accurate nor realistic because we are ignoring many other effects, we wish only to show the over-riding importance of *minimizing the bullet-to-bore contact area*  $A_C$  in controlling bullet friction. The higher contact pressure  $\sigma_{rcp}$  and coefficient of friction  $C_f$  of the copper bullet can be more than compensated by its having a much shorter contact length  $L_C$ .

At muzzle speeds of **2960 fps**, either bullet carries about **4850 ft-lbs** of kinetic energy **KE** at the muzzle. The average base-pressure  $P_{ave}$  (averaged over bullet position in the barrel) can be found by dividing the muzzle energy **KE** by the *Volume of the Bore*  $V_B$  swept by the base of the bullet while being propelled by this spatially averaged base-pressure throughout its bullet-travel distance here of **25.9 inches**.

$$V_B = (\pi/4) * 0.338^2 * 25.9 \text{ in} = 2.3239 \text{ cu. in.}$$

$$P_{ave} = KE / V_B = 12 * 4850 / 2.3239 = 25.04 \text{ ksi}$$

This spatial-average base-pressure  $P_{ave}$  seems reasonable at **49.25 percent** of the accurately estimated peak base-pressure **51 ksi**, and establishes the pressure “peaking ratio” of  $1/0.4925 = 2.0305$  for these bullets in this firing condition.

From an Army technical study reported in Hatcher’s Notebook measuring the total energy disposition in firing a 30-’06 military cartridge, we expect the energy lost to bullet/barrel friction to be about **7.4 percent** of the total energy from the propellant with similar jacketed, lead-cored bullets.

QL calculates the energy efficiency for our 338 load at **29.2 percent**, so the total energy released by the powder is **16,610 foot-pounds** for each shot. Thus, the average partial base-pressure due to friction  $P_{f(ave)}$  for the jacketed bullet should be approximately

$$P_{f(ave)} = 0.074 * 12 * 16,610 / 2.3239 = 6,348 \text{ psi.}$$

and the *peak* friction-equivalent partial pressure  $P_f$  for jacketed bullets should have been about

$$P_f = 6348 / 0.4925 = 12,887 \text{ psi} \quad \text{(Jacketed bullet).}$$

Scaled by our comparative calculations above, we might then expect that for our copper bullet this peak pressure might be about

$$P_f = (23.2 / 29.6) * 12887 = 10,110 \text{ psi} \quad \text{(Copper bullet).}$$

This would then indicate a kinetic energy loss due to friction of just **5.8 percent** with our copper bullets.

This simple comparison indicates that we need not be overly concerned about decreased barrel life or increased barrel cleaning problems due to friction when switching from shooting jacketed, lead-cored match bullets to shooting properly designed and manufactured half-hard copper bullets.

### **8.0 Shot-Start Pressure with Copper Bullets**

Many riflemen are concerned about possibly encountering higher shot-start pressures when switching from firing conventional jacketed, lead-cored rifle bullets to using monolithic copper bullets, or when using rifles having differing chamber throat angles, rifling patterns, and rifling twist-rates. All else being equal, higher shot-start pressures **do** cause significantly higher chamber pressures while producing only marginal gains in muzzle speed.

### 8.1 Engraving of the Rifling Lands into the Bullet

The amount of base pressure  $P_E$  necessary to engrave the rifling into the bullet depends upon (1) the yield stress  $S$  of the bullet material, (2) the throat half-angle  $\Phi$  of the barrel, and (3) the fraction  $k$  of the width of each rifling land divided by the sum of the land-width plus the groove-width for the rifling pattern of the barrel being used. The depth of the rifling grooves is always great enough to produce permanent engraving marks as plastic deformations on the bullet shank or driving band. Thus, the radial stress  $\sigma_r$  needed to impress each land into the bullet depends only upon the yield stress  $S$  of the material being engraved. A radial stress  $\sigma_r$ , equal to or exceeding  $S$ , applied all around the perimeter of the bullet would compress that entire perimeter first elastically and then plastically (and would permanently lengthen that compressed bullet correspondingly).

Let us measure the land and groove widths at one-half the groove depth for our example rifle barrel. According to geometry, the width at half-depth of a single land and groove pair for our rifle barrel would then be  $(\pi/N)*0.3340$  inches, where  $N = 6$  (here) is the integer number of these land/groove pairs in the selected rifling pattern. So, each land/groove pair is **0.1749 inches** in width in this example rifle barrel.

For a match-type rifle barrel with really narrow lands, the land-width fraction  $k$  might be as small as about **0.25**, and the fraction  $k$  might approach **0.5** for a typical military or factory-type hunting rifle. As our rifling lands seem to be about **52 thousandths** wide, we estimate  $k = 0.30$  for our example barrel and will use that value for our calculations here.

The rifling lands act to localize, or concentrate, the radial stress  $\sigma_r$  by a factor of  $1/k$  so that a full-perimeter radial stress of only

$$\sigma_r = k*S = 0.3*40,000 = 12.0 \text{ ksi}$$

is required to engrave the rifling lands fully into the copper bullet.

The base-pressure  $P_E$  required to engrave the rifling lands produces an axial stress  $\sigma_a$  equal to that base-pressure

$$\sigma_a = P_E$$

But this axial stress  $\sigma_a$  produces a radial stress  $\sigma_r$  according to

$$\sigma_r = \mu*\sigma_a$$

where  $\mu$  is Poisson's Ratio of lateral shrinkage for this bullet material under tensile loading.

So that the required engraving base-pressure  $P_E$  is given by

$$P_E = \sigma_a = \sigma_r / \mu = k * S / \mu$$

Or, using our example values for this copper bullet in this rifle barrel,

$$P_E = 0.30 * 40,000 / 0.33 = 36,364 \text{ psi.}$$

This looks pretty fierce, but we have a "mechanical advantage" here which we have not yet considered.

The front slopes of the ends of the rifling lands slope back at a rather shallow throat half-angle  $\Phi$  as measured from the axis of the bore. As a simple tool, this "wedge angle" provides a mechanical advantage given by the trigonometric co-tangent of the throat angle  $\Phi$ . Our custom 338 Lapua Magnum chambering reamer cuts a fairly steep throat half-angle  $\Phi$  equal to **4 degrees**, which has a co-tangent of **14.3**, and which is still quite a significant mechanical advantage. This mechanical advantage would be almost twice as much at **38.2** for a more typical **1.5 degree** throat angle.

So, neglecting friction, the basic shot-start pressure  $P_E$  required to engrave the rifling for our example rifle and our copper bullet is now

$$P_E = \text{Tan}(\Phi) * k * S / \mu = 0.06993 * 36,364 = 2,543 \text{ psi}$$

This basic shot-start pressure  $P_E$  is enough to engrave any reasonable rifling pattern into a monolithic copper bullet to the full depth of the grooves, but without yet allowing either for barrel internal expansion or friction effects.

## **8.2 Compression of the Rear Driving Band to Groove ID**

The rear driving bands of our example copper ULD bullets are made **0.0006-inch** larger in OD than the **0.3380-inch** minimum ID of the grooves in a new (unworn) standard 338-caliber rifle barrel. This extra copper material is elastically compressed within the rifling grooves during the plastic engraving of the nearby lands into that driving band.

The required radial compressive strain ratio  $\epsilon_r$  is given by

$$\epsilon_r = 0.0006 / 0.3380 = 0.001775$$

The radial stress  $\sigma_r$  required to produce this elastic strain ratio  $\epsilon_r$  in our example copper bullet is

$$\sigma_r = E \cdot \epsilon_r = 16,900,000 \cdot 0.001775 = 30.0 \text{ ksi}$$

Now, we can combine  $(1 - k)$  times this elastic radial stress in the grooves of the rifling pattern with  $k$  times the radial stress  $S$  shown above for plastically engraving the lands to find the **total radial stress**  $\sigma_r$  exerted on the copper bullet by the steel throat of the barrel to be

$$\sigma_r(\text{Total}) = (1 - k) \cdot 30.0 + k \cdot S$$

$$\sigma_r(\text{Total}) = 0.70 \cdot 30.0 + 0.30 \cdot 40.0 = 33.0 \text{ ksi}$$

Scaling the barrel radial expansion at peak base pressure  $P$  calculated earlier by using Lamé's Equation, shows that our steel rifle barrel will expand internally here in its throat during rifling engraving by

$$U(r_i) = (33.0/75.6) \cdot 0.000643 \text{ inches}$$

$$U(r_i) = 0.000281 \text{ inches.}$$

Here we are ignoring the tiny amount of barrel expansion due to the shot-start (gas) pressure itself.

The radial expansion of the barrel is linearly proportional to the pressure applied to it internally, whether that pressure is supplied by internal gas pressure or by direct surface-to-surface contact pressure  $\sigma_r$  with our copper bullet. Our example barrel is nearly cylindrical in outer profile here and has the same OD over its throat (**1.24 inches**) as it has **3.2-inches** forward over the point of peak base-pressure application, so that the non-linear portion of Lamé's Equation remains unchanged.

The radial compressive stress relief in the copper bullet caused by this amount  $U(r_i)$  of internal radial barrel expansion is then given by

$$\epsilon_r = U(r_i)/r = -0.000281/0.1690 = -0.0016627$$

$$\sigma_r = E \cdot \epsilon_r = -0.0016627 \cdot 16,900,000 \text{ psi}$$

$$\sigma_r = -28.1 \text{ ksi.}$$

So, the net total radial stress exerted by the copper bullet upon the inside of the steel barrel throat now becomes

$$\sigma_r(\text{Total}) = 0.70*30.0 + 0.30*40.0 - 28.1 = 4.9 \text{ ksi}$$

The total axial stress  $\sigma_a$  needed to produce this total radial stress  $\sigma_r$  is then

$$\sigma_a = \sigma_r/\mu = 4,900/0.33 = 14.85 \text{ ksi.}$$

The throat angle of **4 degrees** produces the same mechanical advantage of **Cotangent(4 degrees) = 14.30**, both for the plastic engraving of the lands and for the elastic compressing of the copper bullet material within the grooves, so that the net combined starting pressure required both for compressing and engraving the copper bullet in the throat  $P_{CE}$ , neglecting friction, becomes

$$P_{CE} = \text{Tan}(\Phi)*\sigma_a = 0.06993*14.85 \text{ ksi} = 1,038 \text{ psi.}$$

Now we need to find how much the friction between the outside of the bullet and the inside of the rifle barrel increases this basic combined shot-start engraving and compressing pressure.

### **8.3 Friction of Copper Bullet in Throat of Steel Barrel**

We showed earlier that a peak base-pressure  $P = 51 \text{ ksi}$  behind our example 338-caliber, base-drilled copper ULD bullet produced a peak radial contact pressure  $\sigma_r = 24.6 \text{ ksi}$ , which is quite similar to the peak radial contact pressure of **22.4 ksi** which would be produced when firing similarly an ideal soft-jacketed match bullet made with a soft core of pure lead.

This earlier calculation even allowed for the internal expansion of the steel rifle barrel caused by that peak base-pressure  $P$  of **51 ksi**, with the added contact pressure  $\sigma_r$  of **24.6 ksi**, for a total of **75.6 ksi** of pressure inside the barrel.

Thus, it would be reasonable to estimate that

$$\sigma_r(t) = (24.6/75.6)*P(t) = 0.3254*P(t)$$

for lower, off-peak base pressures  $P(t)$  as well.

These numbers are basically “smooth-bore” values which do not consider the rifling nor the spin-up of the accelerating bullet. We might term this ratio of pressures  $R_P = 0.3254$  and think of it as an “aggregated” version of Poisson’s Ratio  $\mu$  for this copper bullet.

The total contact area  $A_c$  between the outside of the dual-diameter copper bullet and the inside of the barrel steel can be formulated as

$$A_c = \pi * [(1 - k) * D_g * L_{db} + k * D_b * L_{sh}] / \text{Cos}(Ha)$$

where

$A_c$  = Contact Area in square inches

$k$  = Land-Width Ratio = 0.30 here

$D_g$  = Groove ID of Barrel = 0.3380 inches

$D_b$  = Bore ID of Barrel = 0.3300 inches

$L_{db}$  = Top Length of Rear Driving Band (RDB) = 0.2334 in.

$L_{sh}$  = Total Shank Length (including RDB) = 0.6934 inches

$Ha$  = Helix Angle of Rifling =  $\text{Tan}^{-1}[\pi/n]$  = 9.81 degrees

$n$  = 18.17 = Number of Calibers/Turn of the Rifling.

Thus, the total contact area  $A_c$  for this example is

$$A_c = 0.3949 \text{ square inches.}$$

One could argue that including the cosine projection at the helix angle  $Ha$  into the engraving length is neither necessary nor correct, but it is a very small effect in any case, even for this rather steep helix angle  $Ha$ . Actually, the *widths* of the engraving marks are *reduced* by this same cosine projection effect at the helix angle  $Ha$ , so the contact area  $A_c$  is truly independent of this helix angle.

We merely wished to show that, within reason and contrary to the prevailing perception, the selected rifling twist-rate actually has *no effect* on shot-start pressure  $P_0$ . With an axial second moment of inertia  $I_x$  of just **2.9075 grain-inches<sup>2</sup>**, the initial spin-up torque for this copper bullet requires negligible axial force even at this rather steep helix angle of **9.81 degrees**, especially after taking into account its **5.78:1** mechanical advantage.

Then, the total radial contact force  $F_c$  distributed over this contact area  $A_c$  at any base-pressure  $P(t)$  is given by

$$F_c = \sigma_r * A_c = 0.3254 * P(t) * 0.3949 = 0.1285 * P(t) \text{ pounds}$$

This force  $F_c$  is a distributed normal-direction contact force in the sense ordinarily used to calculate the axial force-of-friction  $F_f$  resisting forward motion of the bullet:

$$F_f = C_f * F_c = 0.36 * 0.1285 * P(t) = 0.04626 * P(t) \text{ pounds}$$

Recall the caveats mentioned earlier about bullet-to-barrel friction not being analytic.

The partial base-pressure  $P_f$  equivalent to this axial frictional force  $F_f$  acting upon the cross-sectional area of the bullet  $A_B$  is just

$$A_B = (\pi/4) * D_g^2 = 0.089727 \text{ square inches}$$

$$P_f = F_f / A_B = 0.04626 * P(t) / 0.089727 = 0.5156 * P(t) \text{ psi}$$

In particular, when  $P(t) = P_{CE} = 1038 \text{ psi}$ , as calculated in the preceding section, the total base-pressure  $P_0$  required for shot-start is the sum of  $P_{CE}$  and  $P_f$

$$P_0 = P_{CE} + P_f = P_{CE} + 0.5156 * P_{CE}$$

$$P_0 = 1.5156 * 1038 = 1573 \text{ psi.}$$

But, now the partial base-pressure needed to overcome friction has increased to

$$P_f = 0.5156 * 1573 \text{ psi} = 811 \text{ psi}$$

and 
$$P_0 = P_{CE} + P_f = 1038 + 811 = 1849 \text{ psi}$$

[etc...]

This calculation would have to be iterated many times to find the final result.

The actual shot-start pressure  $P_0$  must be significantly greater than that first-iteration calculated pressure because we have a mechanical system here incorporating fractional positive-feedback. The base-pressure  $P_f$  needed to overcome bullet friction both depends upon  $P(t)$  itself and is additively combined back with  $P(t)$  to form the shot-start pressure  $P_0$ . That is to say, any friction at all causes compounded additional friction here.

We recognize this as a positive fractional-feedback, closed-loop system with unit amplification  $\alpha = 1.0$  and feedback gain  $\beta = 0.5156 < 1.0$ . The loop-gain  $\gamma$  for such a positive-feedback system when  $\alpha*\beta < 1.0$  is given by

$$\gamma = \alpha/(1 - \alpha*\beta) = 1/(1 - 0.5156) = 2.064$$

So, now we can calculate in closed form the total shot-start pressure  $P_0$ , including friction, as

$$P_0 = \gamma*P_{CE} = 2.064*1038 = \underline{2143 \text{ psi.}}$$

This calculated shot-start pressure  $P_0$  does not include the pressure required to push the bullet from the neck of the cartridge case, which must have occurred prior to bullet engraving with these dual-diameter copper bullets. Unless our chamber has no ball seat, these new bullets cannot be loaded with the rear driving band up against the origin of the rifling, as in “jam seating,” which would increase shot-start pressure by at least whatever pressure is required to push the bullet from its case neck.

This calculated shot-start pressure of **2143 psi** for these monolithic copper ULD bullets is only **59.1 percent** of the default “shot-start” pressure value of **3626 psi (25 MPa)** used in the interior ballistics program QuickLOAD© for soft match-style jacketed, lead-cored rifle bullets, such as our comparative example 250-grain Sierra MatchKing bullet.

This **2143 psi** shot-start pressure is more in line with the QL-recommended value of **1800 psi** when using monolithic soft brass bullets. A shot-start pressure of **6525 psi** is recommended in QL when using hard military or hunting style jacketed bullets or when using more conventionally designed monolithic copper bullets.

Both the peak chamber pressure and the muzzle velocity increase somewhat with any increase in shot-start pressure, and vice-versa, so this lower estimated shot-start pressure might allow larger powder charges or faster burning-rate propellants to be used with these copper bullets.

#### **8.4 Shot-Start Pressure Summary**

We found earlier that, with this same copper bullet and after considering every aspect, the net total radial contact pressure  $\sigma_r$  was **22.9 ksi** at a peak base-pressure  $P$  of **51.0 ksi** acting upon this bullet. We reasoned that this same ratio of pressures  $R_P$

$$R_P = 24.6/(51.0 + 24.6) = 0.3254$$

would provide a good estimation for lower, off-peak base-pressures  $P(t)$  for this same bullet, barrel, and shooting conditions

$$\sigma_r(t) = R_P * \sigma_a(t) = R_P * P(t)$$

We defined the land-width fraction  $k$  and how it is measured at one-half the groove depth. This characteristic of the barrel's rifling pattern is a significant variable in calculating shot-start pressures.

We quantified the elastic compression of the copper driving band material into the grooves of the rifling, along with the plastic engraving of the rifling lands into that material, as

$$\sigma_r = (1 - k) * E * (\Delta r / r) + k * S$$

where  $\Delta r = 0.0003$  inches is the amount of radial compression of the rear driving band required in the rifling grooves, where  $r = 0.1693$  inches here.

We also calculated the internal barrel expansion in its throat due to the radial contact pressure with the copper bullet using Lamé's Equation:

$$U(r_i) = 0.000281 \text{ inches}$$

Internal barrel expansion due to these low shot-start gas pressures alone is negligible.

We formulated the reduction in copper contact pressure attributable to this radial inside barrel expansion as

$$\sigma_r = E * U(r_i) / r$$

and then formulated the net total copper-bullet contact pressure as

$$\sigma_r(\text{Total}) = (1 - k) * E * (\Delta r / r) + k * S - E * U(r_i) / r$$

The axial stress  $\sigma_a(\text{Total})$  which would produce this radial stress  $\sigma_r(\text{Total})$  is formulated as

$$\sigma_a(\text{Total}) = \text{Tan}(\Phi) * \sigma_r(\text{Total}) / \mu = P_{CE}$$

Or

$$P_{CE} = \tan(\Phi) \cdot \sigma_r(\text{Total}) / \mu$$

where  $\Phi$  is the throat half-angle,  $\mu$  is Poisson's Ratio for copper, and  $P_{CE}$  is the base-pressure needed both to *compress elastically* and to *engrave plastically* the copper bullet within the expanding steel barrel, but not yet including friction considerations.

We showed that the friction between the copper bullet and the steel barrel throat constitutes a fractional positive-feedback mechanism, and how to formulate friction calculations for that type of feedback system. The base-pressure  $P_f$  necessary to overcome sliding friction is

$$P_f = C_f \cdot R_P \cdot (A_C / A_B) \cdot P_{CE} = \beta \cdot P_{CE}$$

where

$C_f$  = Coefficient of Sliding Friction

$R_P$  = Ratio of Contact Pressure to Total Internal Pressure

$A_C$  = Contact Area in square inches

$A_B$  = Cross-Sectional Area of the Bore (sq. in.)

$\beta = C_f \cdot R_P \cdot (A_C / A_B) =$  Friction Feedback Factor.

The total shot-start pressure  $P_0$  then initially looks like

$$P_0 = P_{CE} + P_f = P_{CE} + \beta \cdot P_{CE} = (1 + \beta) \cdot P_{CE}$$

But now the "friction" pressure  $P_f$  must increase because that friction now depends upon this augmented value of  $P_0 > P_{CE}$  and this calculation would have to be iterated.

The closed-form expression for the gain  $\gamma$  for this frictional positive-feedback system is

$$\gamma = 1 / (1 - \beta)$$

So the shot-start pressure  $P_0$  can be formulated in closed form as

$$P_0 = \gamma \cdot P_{CE} = P_{CE} / (1 - \beta) = P_{CE} / [1 - C_f \cdot R_P \cdot (A_C / A_B)]$$

Note that, due to the compounding of friction here, the shot-start pressure  $P_0$  does *not* vary linearly with the coefficient of dynamic friction  $C_f$ , whatever its value might be.

We found that the calculated shot-start pressure  $P_0$  of **2143 psi** for these dual-diameter, base-drilled, monolithic copper ULD bullets fired in a very fast-twist match rifle barrel with a rather steep throat angle of **4 degrees** would be significantly less than the **3626 psi** shot-start pressure normally encountered with our comparison soft-jacketed, soft lead-cored match rifle bullet of conventional design. The calculated shot-start pressure for our new monolithic copper bullets is more in line with the **1800 psi** recommendation in QuickLOAD© for firing monolithic bullets made of soft brass.

### **9.0 Copper Bullet Expansion Due to Centrifugal Force**

For the dual-diameter copper bullet, consider a thin annular disc of the rearmost full **0.3386-inch** diameter portion of the rear driving band having a **0.125-inch** diameter hole drilled in its center and rotating about the longitudinal principal axis of the bullet at **6600 revolutions per second**. This is about the fastest rotation rate expected with a 338-caliber projectile as it nears the muzzle of a barrel rifled at **6 inches per turn** at a speed of **3300 fps**. This disc is located within the portion of the copper bullet which would elastically fail first and also where the greatest radial contact pressure due to centrifugal force will occur.

Let us set the outside radius  $R_o = 0.1693$  inches, and the inside radius  $R_i = 0.0625$  inches for this annular disc. The maximum rotation rate  $\omega = 2\pi \cdot 6600 = 41,469$  radians per second as the bullet nears the muzzle.

We have Roark's formulas from mechanics giving the radial stress  $\sigma_r(r)$ , the tangential stress  $\sigma_t(r)$  for any element of this disc positioned at any radial position  $r$  within the disc.

$$\sigma_r(r) = [(3+\mu)/8] \cdot (\rho \cdot \omega^2) \cdot [R_o^2 + R_i^2 - r^2 - (R_o \cdot R_i / r)^2]$$

$$\sigma_t(r) = [(3+\mu)/8] \cdot (\rho \cdot \omega^2) \cdot \{R_o^2 + R_i^2 - [(1+3\mu)/(3+\mu)] \cdot r^2 + (R_o \cdot R_i / r)^2\}$$

where

$$\mu = \text{Poisson's Ratio for Copper} = 0.33$$

$$\rho = \text{Density of Copper} = (2255.8/7000)/(12 \cdot g)$$

$$\rho = 8.3467 \cdot 10^{-4} \text{ slugs/in}^3$$

We know that the radial stress  $\sigma_r(r) = 0$  at  $r = R_o$  and at  $r = R_i$  as boundary conditions, so we do not need to evaluate  $\sigma_r(r)$  here.

We do need to evaluate the tangential stress  $\sigma_t(r)$  at  $r = R_o$  where to see if the bullet fails elastically at this rotation rate.

$$\sigma_t(R_o) = (\rho \cdot \omega^2 / 4) \cdot [(1 - \mu) \cdot R_o^2 + (3 + \mu) \cdot R_i^2]$$

$$\sigma_t(R_o) = (358,842) \cdot [0.0192039 + 0.0130078] = 11.56 \text{ ksi.}$$

Since  $\sigma_r(R_o) = 0$ , and  $\sigma_a(R_o) = -10.8 \text{ ksi}$  (axial compression by base-pressure) at the muzzle, the equivalent three-dimensional von Mises stress  $\sigma_{VM}$  at  $R_o$  on the surface of the copper bullet leaving the muzzle is

$$\sigma_{VM}(R_o) = 0.7071 \cdot \text{SQRT}[\sigma_t^2 + \sigma_a^2 + (\sigma_t - \sigma_a)^2] = 19.37 \text{ ksi.}$$

This level of von Mises stress  $\sigma_{VM}(R_o)$  is safely less than the yield stress  $S$  of **40.0 ksi** for this half-hard copper bullet spinning at its maximum ever likely rate at the muzzle.

We can evaluate the radial displacement  $U(r)$  at  $r = R_o$  to determine the radial expansion of the spinning copper bullet as  $U(R_o)$

$$U(R_o) = \sigma_t(R_o) \cdot (R_o / E)$$

$$U(R_o) = (11,559) \cdot (1.001775 \cdot 10^{-8})$$

$$U(R_o) = 0.0001158 \text{ inches.}$$

Then, the potential radial contact pressure due to centrifugal force  $\sigma_{rcf}$  as the bullet nears the muzzle would be given by

$$\sigma_{rcf} = E \cdot U(R_o) / R_o = \sigma_t(R_o) = \underline{11.6 \text{ ksi.}}$$

The copper bullet is moving at just **1272 fps** and spinning at **2544 revolutions per second (15,984 radians per second)** at the time of peak base-pressure  $P$ . At this time, the radial contact pressure is

$$U(R_o) = (53,312) \cdot (1.001775 \cdot 10^{-8}) \cdot [0.0192039 + 0.0130078]$$

$$U(R_o) = 0.00001720 \text{ inches}$$

$$\sigma_{rcf} = E \cdot U(R_o) / R_o = \underline{1.717 \text{ ksi.}}$$

### **9.1 A Simple Approximation for Rotational Stresses**

The instantaneous distributed centrifugal force **df** acting outward on a thin cylindrical shell element of mass **dm** of the rear driving/sealing band of our copper bullet or of the shank of our example jacketed bullet, at radius **r** from the spin-axis of the bullet, is given by

$$\mathbf{df} = \mathbf{dm} * \mathbf{r} * \boldsymbol{\omega}^2$$

where **ω** is the instantaneous spin-rate of the bullet in radians per second as the bullet is spinning up while traversing the rifled barrel.

The mass element **dm** of the cylindrical shell can be formulated as

$$\mathbf{dm} = \boldsymbol{\rho} * \mathbf{L} * (2\boldsymbol{\pi} * \mathbf{r}) * \mathbf{dr}$$

where **L** is here the axial length of the rear driving/sealing band or of the full-diameter shank of the jacketed bullet.

Combining these expressions we have

$$\mathbf{df} = 2\boldsymbol{\pi} * \boldsymbol{\rho} * \mathbf{L} * \boldsymbol{\omega}^2 * \mathbf{r}^2 * \mathbf{dr}$$

Integrating over the radius **r** from **Ri = 0.0625 inches** to **Ro = 0.1693 inches** to find the total outward-acting centrifugal force **F** exerted by a 338-caliber copper bullet upon the constraining steel walls of the barrel, we have

$$\mathbf{F} = (2/3) * \boldsymbol{\pi} * \boldsymbol{\rho} * \mathbf{L} * \boldsymbol{\omega}^2 * [\mathbf{Ro}^3 - \mathbf{Ri}^3]$$

The tangential stress **σ = F/A**, where the distributed working area **A = 2π\*R\*L**, so

$$\boldsymbol{\sigma} = \mathbf{F}/\mathbf{A} = \boldsymbol{\rho} * \boldsymbol{\omega}^2 / (3 * 12 * \mathbf{g}) * [(\mathbf{Ro}^3 - \mathbf{Ri}^3) / \mathbf{R}]$$

The last two factors in the first denominator are necessary here only if we want to give our copper density **ρ** as a weight per unit volume **2255.8/7000 = 0.32226 pounds per cubic inch** instead of using proper density units of mass per unit volume, such as slugs per cubic inch. The acceleration of gravity **g** is taken to be **32.174 feet per second squared**.

Evaluating the tangential strain ratio **ε = σ/E** at **Ro = 0.1693 inches** at the outer surface of the bullet, with **R = Ro**, we have

$$\boldsymbol{\varepsilon} = 4.4813 * 10^{-13} * \boldsymbol{\omega}^2$$

The spin rate  $\omega$  of the bullet at the muzzle would be **6600 revolutions per second** for our example bullet fired at **3300 fps** from a barrel rifled at an extremely quick rate of **6 inches per turn**. Thus, at the muzzle  $\omega = 2\pi*6600$  **radians per second**, and the maximum strain ratio  $\epsilon$  is

$$\epsilon = 0.0007706$$

If unconstrained, the diameter enlargement due to centripetal force would then be **0.261 thousandths of an inch**, which is small, but not completely insignificant, occurring as a constrained radial stress near the muzzle end of the barrel and as an unconstrained bullet diameter enlargement during subsequent free flight.

Even at **6600 revolutions per second**, the radial stress due to centrifugal force  $\sigma_{rcf}$ ,

$$\sigma_{rcf} = 0.0007706 * E = \underline{13.0 \text{ ksi}}$$

which is only about **10 percent** larger than the more exact calculation above.

This centrifugal enlargement in diameter of monolithic copper bullets varies with the square of bullet spin-rate, which in turn varies linearly with bullet speed down the bore. It starts at zero, has little effect at the time of peak chamber pressure ( $\sigma_{rcf} = 1.7 \text{ ksi}$ ), but peaks rapidly as the bullet nears the muzzle ( $\sigma_{rcf} = 11.6 \text{ ksi}$ ), just as the inertial enlargement and base-pressure ducting enlargement are reducing monotonically to their post-peak minimums. So their total combined effect varies slightly less with bullet position than with any single effect considered separately.

## **9.2 Centrifugal Expansion of Jacketed Bullets**

We can formulate a useful approximation of these centrifugal forces on our jacketed, lead-cored comparison bullet by calculating similarly the outward stress of the spinning lead core with  $R_i = 0$ , adding that to the outward stress of the spinning jacket, and then comparing that sum to the strength of the jacket material. While the jacketed bullet would certainly fail at **6600 RPS**, we can calculate a maximum bullet spin-rate and a corresponding fastest barrel twist rate which the jacketed comparison bullet might survive.

Using the simplified development above, but substituting the properties of lead for copper, we find

$$\sigma_L = \rho_L * \omega^2 * R_L^2 / (3 * 12 * g) \text{ psi}$$

with

$$\rho_L = 2867.8 / 7000 = 0.40969 \text{ pounds/cu. in.}$$

$$R_L = 0.1692 - 0.032 = 0.1372 \text{ inches}$$

$$\sigma_L = 6.65812 * 10^{-6} * \omega^2 \text{ psi}$$

And for the gilding metal jacket

$$\sigma_{GM} = [\rho_{GM} * \omega^2 / (3 * 12 * g)] * [(R_o^3 - R_L^3) / R_o] \text{ psi}$$

with

$$\rho_{GM} = 2240.7 / 7000 = 0.32010 \text{ pounds/cu. in.}$$

$$R_o = 0.1692 \text{ inches}$$

$$\sigma_{GM} = 3.69354 * 10^{-6} * \omega^2 \text{ psi}$$

$$\sigma_{Tot} = \sigma_L + \sigma_{GM} = 1.03517 * 10^{-5} * \omega^2 \text{ psi}$$

But, gilding metal has a yield strength  $S_{GM}$  of only **10,000 psi** and a Modulus of Elasticity  $E_{GM}$  of **17,000,000 psi**.

If we set

$$\sigma_{Tot} = \sigma_{rcf} = S_{GM} = \underline{10.0 \text{ ksi}}$$

we can solve for the maximum spin-rate  $\omega_{Max}$  which might be survived as

$$\omega_{Max} = \text{SQRT}[10,000 / 1.03517 * 10^{-5}] = 31,081 \text{ radians/second}$$

which is 4947 revolutions/second.

At a muzzle speed of **3,000 fps**, the fastest barrel twist-rate  $T_w$  would be

$$T_w = 3000 / 4947 = 0.60647 \text{ feet/turn} = \underline{7.28 \text{ inches/turn.}}$$

The radial expansion  $\Delta r$  of the jacketed bullet at this maximum survivable spin-rate would be

$$\Delta r = R * 10,000 / 17,000,000 = \underline{0.00010 \text{ inches.}}$$

## 10.0 Conclusions

A properly designed 338-caliber bullet can be CNC turned from tough, half-hard UNS C147 copper rod stock and still exceed the performance in interior ballistics of a modern soft-jacketed, soft-lead-core match bullet.

Riflemen are well acquainted with the barrel life and bore cleaning characteristics of these fine traditional match bullets. Key design features

incorporated into these 338-caliber copper ULD bullets which make this interior ballistics performance possible include (1) use of a dual-diameter bullet design having a bore-riding shank ahead of a wide rear driving band, (2) use of a **0.0006-inch** over-diameter rear driving/sealing band, and (3) base-drilling these copper bullets to a suitable depth of **0.396 inches** using a suitable drill diameter of **0.125 inches**.

With these design features incorporated, gas blow-by at peak base pressure is minimized with copper bullets, making for higher muzzle speeds and better uniformity of those muzzle speeds. Years of experience firing jacketed lead-cored bullets, which routinely “slug-up” to match the ID of pressure-expanded rifle barrels, did not prepare us to appreciate fully the difficulty which non-deforming half-hard copper bullets would encounter in sealing the peak base pressures they would encounter in rifle interior ballistics.

Even a heavy, cylindrical match 338-caliber barrel can expand internally by **1.3 thousandths of an inch** at typical peak base-pressures. Relaxing the initial elastic compression of the over-diameter rear driving band allows the half-hard copper 338 bullet to expand by **0.6 thousandths**. The remaining **0.7 thousandths** of barrel ID expansion is less than the maximum elastic expansion of **0.8 thousandths** for this half-hard copper bullet. The separately analyzed elastic expansions in copper bullet diameter due to (1) inertial acceleration **0.34 thousandths**, (2) Base-drilling **0.65 thousandths**, and (3) centrifugal expansion **0.034 thousandths** total **1.02 thousandths of an inch**, allowing the calculated **24.6 ksi** internal gas-sealing pressure of the copper bullet against the steel interior of the rifle barrel.

We determined that the radial contact pressure between the outside of the bullet and the inside of the barrel at peak base-pressure was a suitable metric for comparison of gas-sealing abilities. Our copper 338-caliber ULD produces **24.6 ksi** of radial contact pressure at peak base-pressure compared to **22.4 ksi** for the 250-grain Sierra MatchKing bullet.

These dual-diameter copper bullets require only an estimated **52.2 percent** of the shot-start pressure expected in QuickLOAD© to be required of conventional soft-jacketed match bullets like our 250-grain Sierra MatchKing comparative example bullet here. Copper bullets of more

conventional designs often encounter chamber pressure increases attributable to their much higher shot-start pressure requirements.

Friction between these copper bullets and the barrel steel is estimated to be only about **75 percent** of the friction encountered when firing conventional jacketed, lead-cored match bullets. This is mainly attributable to the much reduced area of contact for the rear driving band design of the dual-diameter copper bullets. So, barrel life and bore cleaning requirements with these new copper bullets should be comparable to what we expect with conventional bullets.

While this elementary study cannot produce firm numerical values due both to its over simplification and to the large variances to be expected among rifle bullets and barrels, at least we can see why some copper bullet designs will perform better than others in the rifle barrel.