

Mils/MOA & The Range Estimation Equations

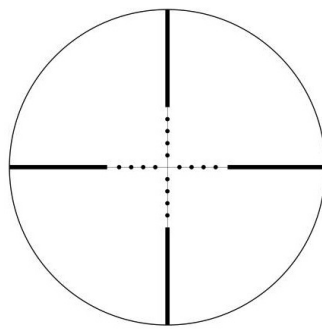
An In-Depth Study for Shooters

ROBERT J. SIMEONE
2nd Edition
December 6, 2016

Overview

Many shooters use scopes that have reticles etched in mils or MOA to take measurements of targets. They then input those measurements into simple equations to estimate the distances to those targets. Based on those estimated distances, they then adjust their scopes' reticles using adjustment controls also calibrated in mils or MOA.

But what exactly is a mil or a MOA and how did they derive the range estimation equations? This paper will try to answer those questions so that shooters will have a better understanding of mils and MOA and the range estimation equations. Hopefully you will become a better-educated and knowledgeable shooter.



Contents

1	Introduction	1
2	Mils	1
3	The Mil Range Estimation Equation	14
4	Minute of Angle (MOA)	21
5	MOA Range Estimation Equation	21
6	Shooter Minute of Angle (S-MOA)	32
7	Conversion Between Units	36
8	Distance Equations	39
9	Quick Reference Guide	41
10	Comparison of Angles	42
11	The Big Picture	43
12	Acknowledgements	44

1 Introduction

Many shooters use scopes that have reticles etched in mils or MOA to take measurements of targets. They then input those measurements into simple equations to estimate the distances to those targets. Based on those estimated distances, they then adjust their scopes' reticles using adjustment controls also calibrated in mils or MOA. But what exactly is a mil or a MOA and how did they derive the range estimation equations? This paper will try to answer those questions so that shooters will have a better understanding of mils and MOA and the range estimation equations. Hopefully you will become a better-educated and knowledgeable shooter.

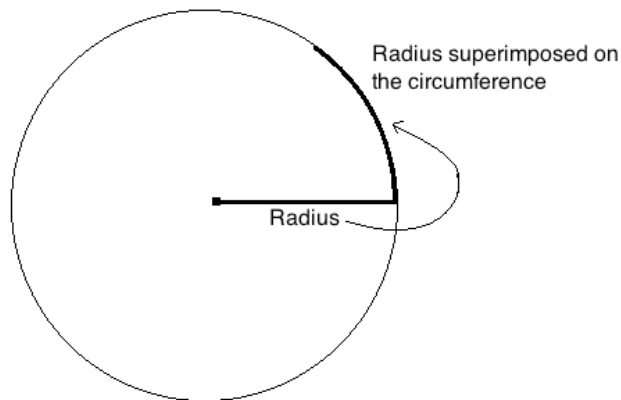
Note: The discussions that follow sometimes involve math and math equations. Although simple, most times they are not a necessary requirement for the understanding of the subject. Therefore, often when math comes into the discussion, I will offer the option of skipping over that part by separating the math section with two bold black lines (as shown above and below) and give you a page number and an (→) to indicate where you can pick up the subject matter again.

2 Mils

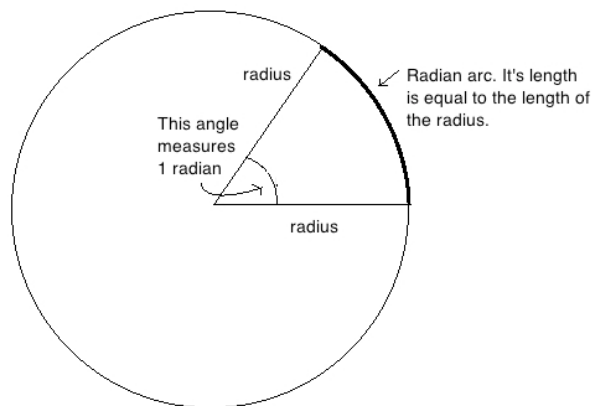
Mils have been used for many years as a mathematical tool to estimate the distance to targets. Used early on by artillery units, naval gunfire units, and most recently by snipers, they are a valuable tool in estimating distances for accurate shots. But many people are unfamiliar with the term "mil". So, what is a mil?

When shooters use the term "mil", what they are really talking about is a shortened form of the word milliradian, which is a mathematical unit of angular measurement of a circle. The "Milli" part of the word "milliradian" is Latin and it means a thousandth (1/1,000th). For example, a millimeter is one one-thousandth (1/1000th) of a meter. In the same manner, a milliradian (usually shortened to just "mil") is one thousandth (1/1000th) of a radian.

So what is a radian? A radian is an angle based on the properties of a circle. To use the natural physical properties of a circle to define what a radian is: (1) make a circle of any size, (2) superimpose the radius of the circle onto the circumference of the circle (below).



The portion of the circumference “covered” by the superimposed radius is called the “arc” or “radian arc”. Remember, **the arc length is the exact same length as the radius**. Now connect the other side of the arc to the center with another radius, creating a pie shape. The central angle formed by this pie shape is called a “radian” (below).



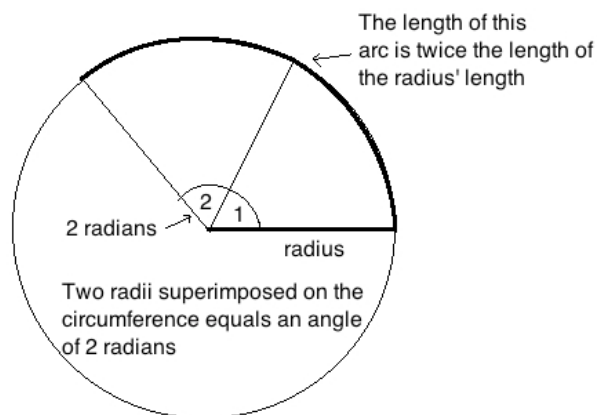
Radians are units of angles just like the more familiar degrees are; they’re just an alternative way of measuring them. They use the actual radius of a circle to define the angle instead of some arbitrary man made number like 360° . Believe it or not, 360° in a circle is an arbitrary man made

number. Radians have advantages in math, science, and engineering that make them much more simple and advantageous to use (although those advantages are beyond the scope of this paper).

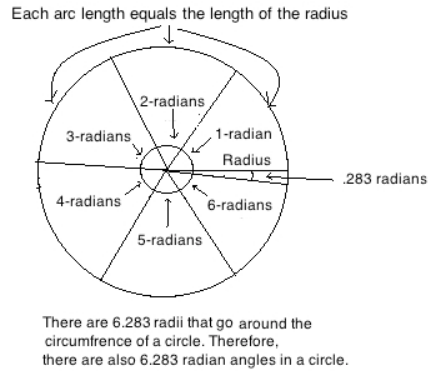
Note: A “radian” or “1 radian” means the same thing. It’s just like saying “a degree” or “1 degree”.

The example above showed what 1 radian is. **One-radian is based on the arc length being exactly the same length as the length of the radius.** But the arc length doesn’t always have to be the same length as the radius’ length; it can be longer or shorter. In those cases, we’ll have radian angles that are also larger or smaller than a 1 radian angle. Let’s look at some examples of what multiple radians, as well as fractions of radians, look like to show you what I mean.

Two radians would be created by two radii (the plural of radius) superimposed on the circumference of a circle, creating an arc length twice the size of the radius’s length, and would look like this:



If we continue adding radii to the circumference and go around the entire circle, it would take approximately 6.283 radius lengths to go around the full circumference of a circle. Therefore, there are 6.283 (approximately) radians in a full circle (below).



Note: If you'd like to see the derivation mathematically, see below; otherwise continue below the second black line this page.

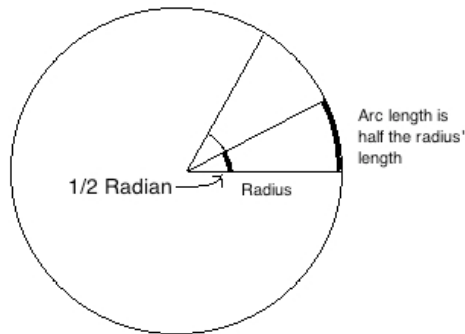
The equation for the circumference of a circle is: $C = 2\pi r$. If you notice in the equation, the circumference is always going to equal the radius' length (no matter how long it is) multiplied by the constant 2π . Since (π) is approximately equal to 3.14159, then:

$$C = 2\pi r = 2 \times 3.14159 \times r = 6.283 \times r.$$

This means that if you take the radius' length (no matter what length it is) and multiply it by 6.283, you will go all the way around the circumference of a circle. Since the "radian angle" is directly related to the number of radius lengths superimposed on the arc, and since there are always 6.283 of them, then that means there are also 6.283 radians (the angle) in a circle. On a side note, since there are 6.283 radians and 360° in a full circle, there are: $360 \div 6.283 = 57.3^\circ$ per radian.

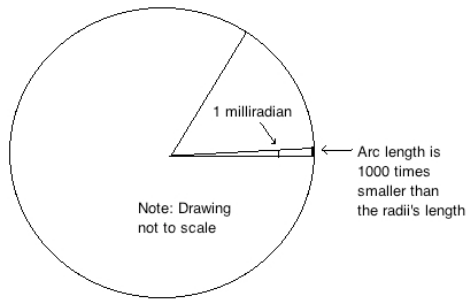
(→)

Going the other way, 1/2 radian means the arc length that is superimposed on the radius is only 1/2 the size of the radius's length, thus making the angle only 1/2 radian (below).



A fundamental point to re-emphasize, as you can see in the examples given so far, is that the angle is directly related to the arc length. That is, **the length of the arc directly dictates the size of the angle**. This is important to remember, especially when you look at the next example below.

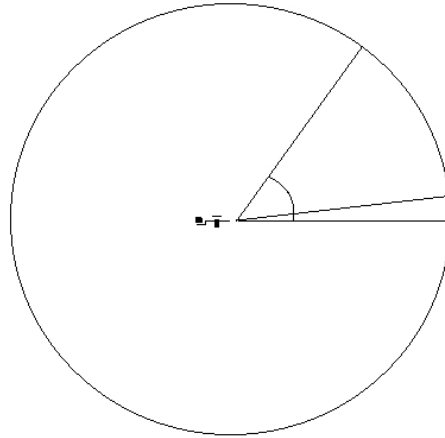
If the arc length superimposed on the circumference is 1,000 times smaller than the radius' length, the angle created is 1,000 times smaller than a radian, which means the angle is $1/1000^{\text{th}}$ of a radian in measurement. As discussed on page 1, since it's 1,000 times smaller than a radian, it's called a milliradian, or a "mil".



Note: We will now use the word "mil" or "mils" mostly from here on out.

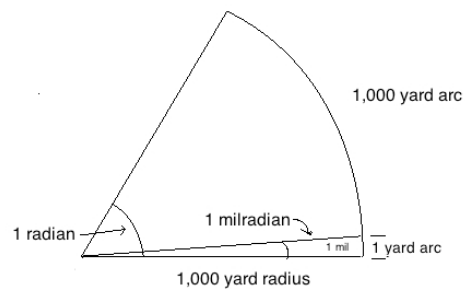
So that is what a mil is; it's an angle 1,000 times smaller than 1 radian, where the arc length of that angle is 1,000 times shorter than the radius' length.

At this point in the discussion, I want you to now visualize that a shooter is at the center of a circle, because theoretically, that is really where a shooter is when using mils to measure angles (below).



Let's continue to look at some further examples of mils, this time with actual numbers in them, to get a better understanding of mils and how they can be used.

If the radius of a circle is 1,000 yards long, we know an angle of 1 radian has an associated arc length also 1,000 yards. It then follows that an angle of 1 mil, one thousand times smaller in angular measurement, must have an associated arc length one thousand times smaller than 1,000 yards, which would be 1 yard (below).

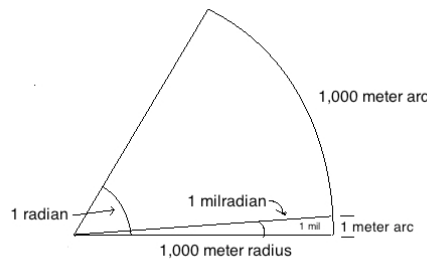


That is, at 1,000 yards, 1 mil has an arc length that is 1 yard in height. Or to put it another way:

“At 1,000 yards, 1 mil equals 1 yard”.

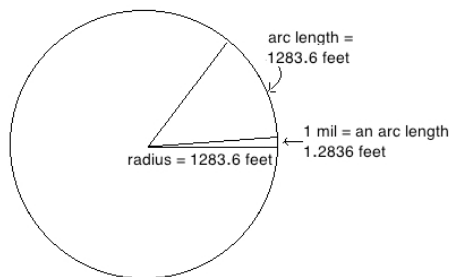
For many shooters, that is an expression that you might have heard before. Now you know where it comes from and why.

Using the same process we can see that if the radius were 1,000 meters, a 1 mil angle would have an arc length that is 1 meter in height:



Therefore, "at 1000 meters, 1 mil equals 1 meter".

As you can see, it's very easy to visualize the relationship between the angle of 1 mil and the height of the associated arc when using easy numbers like 1,000. But it works for any number. For example, if the radius were 1283.6 feet in length, then a 1 mil angle would have an arc 1,000 times smaller, or 1.2836 feet in length:



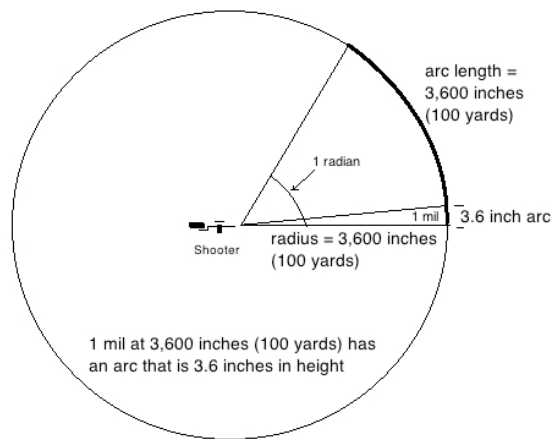
Let's look at two more common distances that shooters frequently use and are familiar with to not only solidify our understanding of mils, but also to show you where two more common shooting phrases come from.

American shooters are very familiar with the distance of 100 yards. But for this example of 100 yards, I want to first convert it to inches (you'll see why I did this shortly): A 100 yard shot converted to inches is:

$$36 \text{ inches per yard} \times 100 \text{ yards} = 3,600 \text{ inches.}$$

As you can see in the picture below, 1 radian at a radius of 3,600 inches has an associated arc length of 3,600 inches long. It then follows that 1 mil, 1,000 times smaller in angular measurement, has an associated arc length that is 1,000 times smaller than 3,600 inches, which would be 3.6 inches. Or put more simply:

“At 100 yards (3,600 inches), 1 mil equals 3.6 inches.”

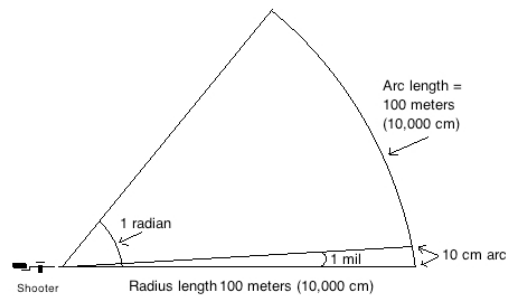


If we were talking about meters, with each meter having 100 centimeters (cm) in them, that means at 100 meters we have 10,000 cm (below):

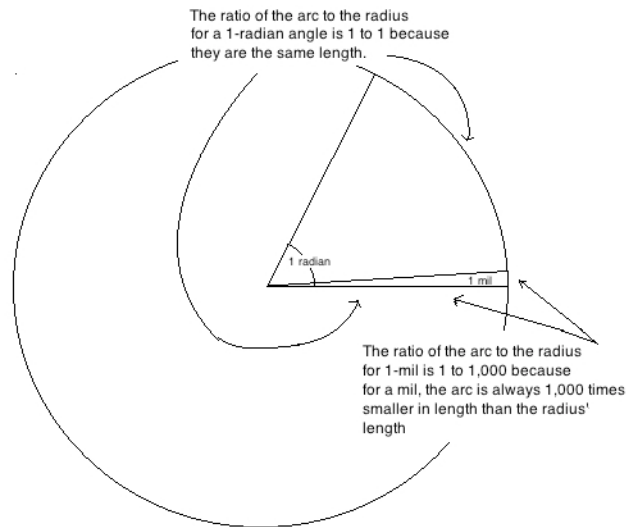
$$100 \text{ meters} \times \frac{100 \text{ cm}}{1 \text{ meter}} = 10,000 \text{ cm.}$$

Therefore, every mil would have an associated arc length 1,000 times smaller than 10,000 cm, which would be: $10,000 \text{ cm} \div 1,000 = \mathbf{10 \text{ cm}}$. That is:

“At 100 meters, 1 mil equals 10 cm”.



At this point I want to emphasize something. We know that the arc length and the radius' length for a 1 radian angle are always equal, meaning the ratio of the arc to the radius for a 1 radian angle is always 1 to 1. We also know that for a 1 mil angle, as shown in the examples above, the arc length is always exactly 1,000 times smaller than the radius' length. That means the ratio of the arc's length to the radius' length for a 1 mil angle is always going to be 1 to 1,000 (see picture below). Later on, this fact will help us come up with a range equation.



Let's define that mathematically and get an equation for the arc length for 1 mil:

The ratio of the length of the arc to the length of the radius for 1 mil is:

$$\frac{\text{arc}}{\text{radius}} = \frac{1}{1,000}.$$

Solve for the arc length by cross multiplying:

$$\text{arc} = \frac{\text{radius}}{1,000}.$$

Therefore for 1-mil:

$$\frac{\text{radius}}{1,000} = \text{arc length}.$$

For example, plug the various radius lengths (below) in the equation (above):

1 mil at a radius of 3,600 inches (100 yards) equals an arc length of 3.6 inches.

1 mil at a radius of 100 meters (10,000 cm) equals an arc length of 10 cm.

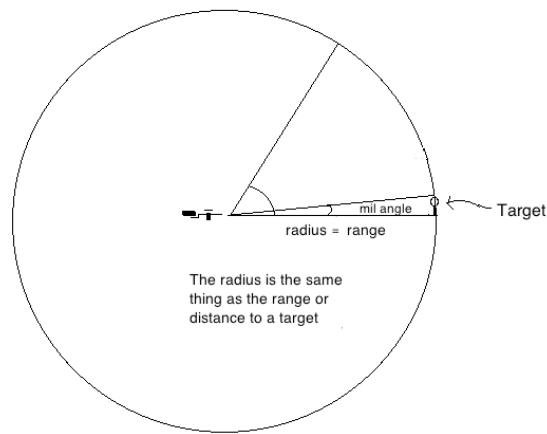
1 mil at a radius of 1,000 yards equals an arc length of 1 yard.

1 mil at a radius of 1,000 meters equals an arc length of 1 meter.

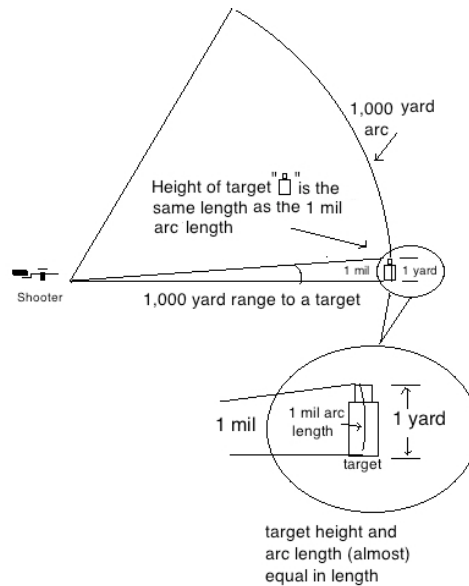
1 mil at a radius of 1,389 feet equals an arc length of 1.389 feet.

1 mil at a radius of 2,500 miles equals an arc length of 2.5 miles.

I'd like to point out two observations. As you can see in the previous pictures, from a shooter's perspective, the radius is the same thing as the range or distance to the target. Therefore, from here on out, I will mostly be using the terms "range" or "distance to the target" instead of the radius (below).



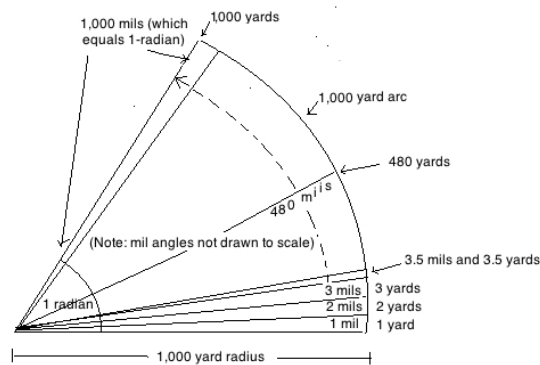
Also, as you can see in the picture above, the arc length can be thought of as the height of the target. For example, as discussed previously, 1 mil at 1,000 yards covers an arc that is 1 yard in height (see page 6 again). Now think of the arc height as the height of a target (below).



Note: Because targets are measured in straight lines from top to bottom, they are slightly shorter in length than the curved arc of a circle. At shooting distances and

at these small angles, for all practical purposes we can consider the difference in length to be negligible and its effects on the math are insignificant.

Thus far we've been only talking about 1 mil and what that equates to in terms of arc lengths or heights of targets at certain distances. That was to help us get an understanding of what a mil is and how it relates to a circle. Now that we have this understanding, it's easy to picture what multiple mils and their arc lengths look like. In the picture below, I chose the 1,000-yard range example to show you what various lengths of mils would look like.



As you can see in the picture above, 1 mil equals or covers an arc height of 1 yard, 2 mils equals or covers an arc height of 2 yards; 3 mils covers an arc height of 3 yards; 3.5 mils covers 3.5 yards; 480 mils covers an arc 480-yards high etc., all the way up to 1,000 mils, in which case 1,000 mils would cover an arc length or arc height of 1,000 yards. Also note that at 1,000 mils, the arc length is equal to the radius' length (in this case, 1,000 yards). What angle has the arc length equal to the radius' length? One radian (see page 2 again).

Therefore, almost exactly like the equation on page 10 for 1 mil, the ratio of the arc to the radius for any mil number is:

$$\begin{aligned} \text{The ratio of the arc to the radius for 1-mil is: } \frac{\text{arc}}{\text{radius}} &= \frac{1}{1,000}. \\ \text{The ratio of the arc to the radius for 2-mils is: } \frac{\text{arc}}{\text{radius}} &= \frac{2}{1,000}. \\ \text{The ratio of the arc to the radius for 3-mils is: } \frac{\text{arc}}{\text{radius}} &= \frac{3}{1,000}. \\ \text{The ratio of the arc to the radius for } x\text{-mils is: } \frac{\text{arc}}{\text{radius}} &= \frac{X}{1,000}. \end{aligned}$$

Cross multiplying to solve for the arc length:

$$\text{arc} = \frac{\text{radius} \times X}{1,000}.$$

That is, the equation for the arc length for any mil number (X) is the range (or radius) times the number of mils, divided by 1,000 (below):

$$\text{For any mil number } X : \frac{\text{range} \times X \text{ mils}}{1,000} = \text{arc length (or height of target)}.$$

For example, if the range is 1,000 yards and X = 3 mils then we have:

$$\frac{1,000 \text{ yards} \times 3 \text{ mils}}{1,000} = 3 \text{ yards}.$$

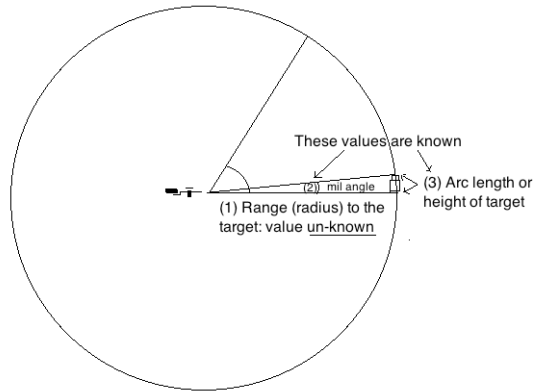
Notice in the equation above, if the radius (or range) increases for a particular mil number, the height of the arc increases by the same magnitude. That is, if you double the range length, then the arc length also doubles. If you triple the range length, the arc length triples etc.

So, in the example above, if X = 3 mils but the range increases to 2,000 yards, then:

$$\frac{2,000 \times 3 \text{ mils}}{1,000} = 6 \text{ yards}.$$

Now that we have this knowledge of mils and how they relate to a circle, how can we use this information to help us figure out the unknown distance to a target?

If you've noticed, we've been discussing three variables up to this point: (1) the range to the target (previously called the radius), (2) the angle in mils and (3) the height of the target (previously called the length of the arc). For shooters, the unknown variable will be (1), the range to the target (below).



For the math in the range estimation equation to work, we must know any two of the three values in order to figure out the third. For shooters, the unknown variable as discussed is the range to the target. Therefore, we must know the mil angle and the height of the target beforehand to figure out the range to the target.

3 The Mil Range Estimation Equation

With the tools of math and the inherent properties of the mil angles, it is easy to come up with a simple equation to get the range to a target.

Note: If you'd like to skip the math, please jump to below the black line on the next page.

Recall from the previous page the equation for figuring out the height of an arc (or the height of a target) for any mil number "X" is:

$$\frac{\text{range} \times X \text{ mils}}{1,000} = \text{arc length (or height of target)}.$$

Let's simplify this to make it easier to read: Let "R" equal the range, let "X" equal the number of mils and let "H" equal the arc length or the height of the target. The equation now looks like this:

$$\frac{R \times X}{1,000} = H.$$

Recall from the statement on the previous page, "For shooters, the unknown variable is the range to the target". That means in the equation above, we know the value of X and H, but we don't know R.

Therefore, we must solve for R . We do that by performing cross-multiplication to put the known values (the height of the target (H), and the mil angle (X)) on the same side.

$$\frac{R \times X}{1,000} = H$$

multiply both sides by 1,000

$$1,000 \frac{R \times X}{1,000} = H \times 1,000$$

$$R \times X = H \times 1,000$$

divide each side by X

$$\frac{R \times \cancel{X}}{\cancel{X}} = \frac{H \times 1,000}{X}$$

$$R = \frac{H \times 1,000}{X}$$

Moving R over to the right hand side to make the equation look more familiar:

$$\frac{H \times 1,000}{X} = R.$$

(\rightarrow)

We now have the equation to determine the range or distance to a target in its simplest form; the height of a target (H) multiplied by 1,000 divided by the number of mils (X) equals the range or distance to a target.

$$\frac{H \times 1,000}{X} = R.$$

Let's see how it works using some previous examples that we already know the answer to.

Say we are shooting at an object that is 1 yard in height and it covers 1 mil in our scope. How far away is it? Plug the numbers in the equation and solve:

$$\frac{1 \text{ yard} \times 1,000}{1 \text{ mil}} = 1,000 \text{ yards.}$$

The distance to the target is 1,000 yards away, just as we knew it should be (see page 6 again).

Here's another example that we already know the answer to (see page 8):

You see an object that you know is 3.6 inches in height and you see it measures 1 mil in your scope. How far away is the object? Plug the numbers in the equation and you get:

$$\frac{3.6 \text{ inches} \times 1,000}{1 \text{ mil}} = 3,600 \text{ inches.}$$

The 3.6-inch object covering 1 mil is 3,600 inches (100 yards) away, the same answer we got using a different method on page 8.

At this point I'm now going to make a slight change to the verbiage and labeling of the basic range equation. It was a smooth transition from the word "radius" to the word "range", as they sound similar and it kept things a little less confusing. But now I'm going to call and label the range (R) the distance (D) to the target, as that is the way the equation is mostly labeled in publications and is a little more familiar word for most non-military shooters. Therefore, the basic equation now looks like this:

$$\frac{H \times 1,000}{X \text{ mils}} = D \text{ (distance).}$$

A very important point to make at this time is that **the units of measurement you use for the height of an object are the same units you will get for the distance**. That is, if you measure the height of an object in inches, the distance you get will also be in inches. If you measure the height of an object in centimeters, the distance you get will also be in centimeters etc. For example, if you're using **meters** for the height that you measure an object in, then the equation will look like this and your distance result will be in **meters**:

$$\frac{H \text{ meters} \times 1,000}{X} = D \text{ meters.}$$

If you're using **yards** for the height of the object you're shooting at, the distance will be in **yards** and the equation looks like this:

$$\frac{H \text{ yards} \times 1,000}{X} = D \text{ yards.}$$

I will use the equation for yards (above) to show how we can manipulate it to make it a little more user-friendly. The equation for yards is simple but not always practical. Let me show you an example of what I mean. Say there is a target of known height, 1/3 of a yard, and it covers 2.25 mils in a scope. How far away is it?

$$\frac{\frac{1}{3} \text{ yard} \times 1,000}{2.25 \text{ mils}} = 148 \text{ yards.}$$

It's 148 yards away. But how many people measure the height of small targets in fractions of a yard? It's much easier to use inches. Yet if you use

inches in the equation for the height, then the distance you get to it will also be in inches, and that's not practical either. For example, if you used inches for the height of the target in the example above, then the distance you would have gotten would also have been in inches:

$$\frac{12 \text{ inches} \times 1,000}{2.25 \text{ mils}} = 5,333 \text{ inches.}$$

As you can see, inches are not a very practical way to measure long distances. Therefore, let's use a simple conversion in the equation that allows us to measure the heights of targets in **inches** yet gives us the distances to them in **yards**:

$$\frac{H \text{ inches}}{36 \frac{\text{inches}}{\text{yard}}} = \frac{H \cancel{\text{inches}}}{36} \cdot \frac{\text{yard}}{\cancel{\text{inches}}} = \frac{H \text{ yards}}{36} = X \text{ yards.}$$

That is:

$$\frac{H \text{ in inches}}{36} = H \text{ in yards.}$$

A height measured in inches divided by 36 equals the height in yards. Here are two examples:

$$\frac{36 \text{ inches}}{36} = 1 \text{ yard,} \quad \frac{12 \text{ inches}}{36} = 1/3 \text{ yard.}$$

This conversion substituted into the distance equation allows us to measure the heights of targets in inches but gives us the distances to them in yards (below).

Note: To skip the math, please jump to below the black line on the next page.

$$\frac{H \text{ yards} \times 1,000}{X \text{ mils}} = D \text{ yards}$$

$$\text{Then, using the fact that } \frac{H \text{ in inches}}{36} = H \text{ in yards,}$$

$$\frac{\frac{H \text{ in inches}}{36} \times 1,000}{X \text{ mils}} = D \text{ yards.}$$

We can simplify this equation by using some properties of math:

$$\frac{\frac{H \text{ in inches}}{36} \times 1,000}{X \text{ mils}} = D \text{ yards}$$

$$\frac{\frac{H \text{ in inches} \times 1,000}{36}}{X \text{ mils}} = D \text{ yards}$$

$$\frac{H \text{ in inches} \times \frac{1,000}{36}}{X \text{ mils}} = D \text{ yards.}$$

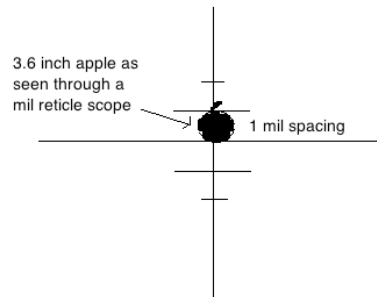
(→)

Simplified you get this equation:

$$\frac{H \text{ in inches} \times 27.8}{X \text{ mils}} = D \text{ yards.}$$

This new equation let's us use **inches** for measuring the height of an object yet at the same time, because of the conversion we used, gives us the distance to it in **yards**. This is very advantageous, particularly to American shooters, who are used to these units of measurement for heights and distances.

Let's look at an example of how this new distance equation works. Say we're shooting at an apple that we know is 3.6 inches in height. Looking through our scope, we see that it fit's exactly between a 1-mil spacing in our reticle:



Plugging the numbers into our equation and then solving, we get:

$$\frac{H \text{ inches} \times 27.8}{X \text{ mils}} = D \text{ yards}$$

$$\frac{3.6 \times 27.8}{1 \text{ mil}} = D$$

$$D = 100 \text{ yards.}$$

Look familiar? Look at the picture on page 8 again. One mil at 100 yards equals an object 3.6 inches in height. We got the same answer both times, just using different methods. One involved a conceptual picture (page 8) and the other used a simple equation (above). Also, unlike the example on the top of page 17 where we got the distance in inches, this time we got the distance in yards.

What if the apple measured only a $\frac{1}{2}$ mil (.5 mils) in the spacing on our scope's reticle; what would the distance to the apple be?

$$\frac{3.6 \times 27.8}{.5 \text{ mils}} = 200 \text{ yards.}$$

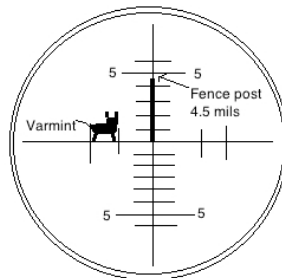
That makes sense, since the apple would appear to be smaller in our scope because it is farther away, thus covering only $\frac{1}{2}$ mil in our scope at 200 yards.

The beauty of the equation and the mil spacing on the scope is that estimating distances can be done quickly and easily with a simple calculation.

Let's look at one more example and show how a shooter really uses all the knowledge we've learned so far.

There is a varmint that has been eating some garden vegetables and we want to end that situation. We don't know the height of the varmint, but do see that he's near a fence post that we put up a few years ago and know is 48 inches in height above the ground. Looking through the scope, we see that the fence post covers 4.5 mils in the scope. How far away is that varmint?

(Scope etched in mils)



Plugging the known numbers into the distance equation:

$$\begin{aligned} \frac{H \text{ inches} \times 27.8}{X \text{ mils}} &= D \text{ yards} \\ \frac{48 \text{ inches} \times 27.8}{4.5 \text{ mils}} &= D \text{ yards} \\ D &= 296 \text{ yards.} \end{aligned}$$

Using the distance calculated by the range equation (roughly 300 yards) we can now adjust the scope for a 300 yard shot to the varmint.

Note: That was the last time the varmint ate the vegetables.

Many people around the world don't measure objects in inches or distances to targets in yards though. That's not a problem; the equation can be adjusted for any combination of units that you desire to use for measuring the heights of targets and the units you want for the distance to them, just like we did for the "inches to yards" example above. I won't go through the math again, but it simply involves just plugging in conversions between units (like we did above for inches to yards; $H/36$) and getting the new results. You can see page 39 at the end of this paper for the distance equations in mils using various units of measurements.

Summary of mils

Mils are not a familiar way of measuring angles for most people. As a matter of fact, most non-shooters or non-military people probably have never even heard of them before. But because of the "natural" way they are defined and the advantages that come with that, it is easy to come up with a simple equation for estimating the distance to a target resulting in it being a valuable tool for shooters.

One last and important note on mils before we move onto the next unit of angular measurement is that the mils that we have talked about and studied so far are actually based on the real physical properties of a circle. They are a real trigonometric unit of angular measurement and are "true mils". That begs the question then, "what other kind of mils are there?"

Remember on page 4 when I said there were approximately 6.283 radians in a circle? Since each of those radian angles contains 1,000-milliradians (or 1,000 mils) that means there are approximately 6,283 mils in an entire circle ($6.283 \times 1,000$). The unevenness of that number provided problems for artillery units, as they wanted to simplify certain calculations with nice rounded numbers. Therefore, they manipulated the angle slightly to make the number of mils in a circle more rounded. Therefore, some military units as well as other countries use a slightly different "artificial" number for the number of mils in a circle. For instance, some artillery and naval gunfire units use 6,400 mils in a circle; the Russians use 6,000 mils in a circle etc. Therefore, their reticles and equations are slightly different. It's important to note that most, **but not all**, scopes are calibrated in "true" mils. Therefore, you might need to do some research into the kind of mils your scope is calibrated in and the slightly different equations they would use.

4 Minute of Angle (MOA)

In this section, we're going to talk about the other unit of angular measurement that shooters use, "minute of angle", abbreviated as MOA. MOA are based on degrees, which unlike mils, most people are familiar with. They are used quite often in shooting and have in fact been used by shooters for many more years than mils have.

Note: "Minute of angle", "MOA" or "Minute" all mean the same thing.

Early scopes mostly used just plain cross hairs in their reticles. Later on they were made so that the reticle could be adjusted for windage and elevation and the units they used were mostly in the units of MOA, usually in $\frac{1}{2}$ or $\frac{1}{4}$ MOA adjustment per click. We still see that today.

Starting in Vietnam because of the military requirement for ranging for snipers, scopes started to come equipped with mil reticles. Interestingly though, the scope manufacturers and the military still kept their adjustments for windage and elevation in MOA, an entirely different unit of angular measurement than mils. (One can equate that to the speed limit on a highway being in miles per hour but the odometer in your car being in kilometers per hour. There is a conversion that you must do to equate the two). Only years later did scope manufacturers (by popular demand, I'm sure) finally start making scopes in the same units of angular measurement for both their reticles and adjustments; either with a mil reticle and mil adjustments, or a MOA reticle and MOA adjustments.

There is another slightly different version of MOA that shooters use and that some riflescopes are calibrated in. It's close in measurement to MOA (whom some call "true" MOA) in terms of measurement, but not exactly equal to it. It is referred to as "**inch per 100 yards**" (IPHY) or what some shooters call, "**shooter's MOA**" (abbreviated S-MOA). Similar to the discussion at the end of the mil section, this unit was made up so that measurements were more "rounded". Unlike reticles in mils that mostly use "true" mils in their reticles, many scope manufacturers do use S-MOA instead of "true" MOA in their reticles and/or adjustments. Therefore, we will discuss and analyze both types of MOA.

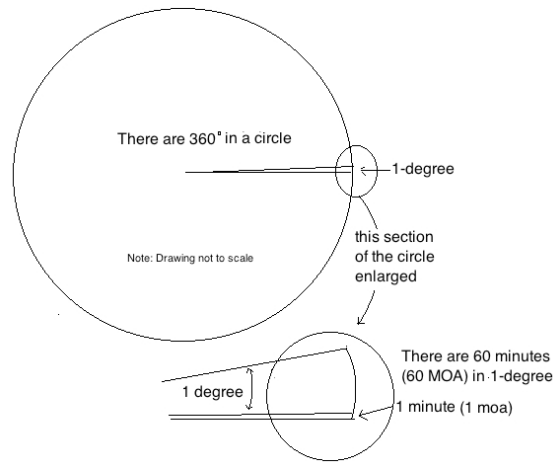
5 MOA Range Estimation Equation

There are 360 degrees (360°) in a circle.

Note: Why there are 360 of them is based on systems used long ago by ancient mathematicians, but they are not based on the actual physical properties of a circle

like radians are. Because degrees are man-made and don't have the "natural" properties that radians do, coming up with a distance equation for MOA will be a little different than it was for mils.

Each degree contains 60 minutes or has 60 minutes of angle (MOA) in them. Another way of saying that is: $1/60$ th of a degree ($.01666667^\circ$) = 1 MOA.



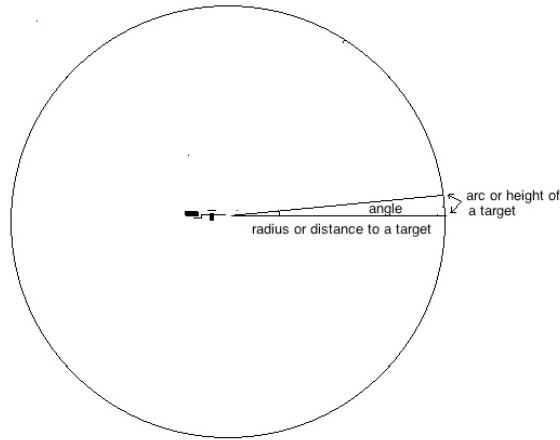
This means that there are a total of 21,600 minutes of angle in a circle:

$$360^\circ \times \frac{60 \text{ minutes}}{1^\circ} = 360 \cancel{^\circ} \times \frac{60 \text{ minutes}}{1 \cancel{^\circ}} = 21,600 \text{ minutes.}$$

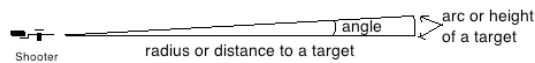
Since there are 21,600 MOA in a circle compared to 6,283 mils in a circle, MOA are a much finer unit of angular measurement than mils are.

In the discussion on mils we took a meticulous step-by-step process starting with the origin of the angle and the natural relationship of the arc and the radius, to a nice easy ratio that produced a simple equation. Here we won't have to do that because there isn't any natural relationship. Therefore, we can get right into figuring out the distance to a target if given an angle in degrees (remember, MOA are fractions of degrees) and the height of the target. There are several ways to do this and I chose the method that I thought was the easiest.

Picture a shooter at the center of a circle looking at a target (below):

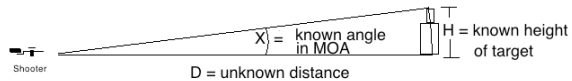


Let's disregard most of the circle so that we can concentrate on just the small pie shape again.

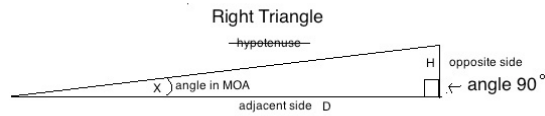


Just like when using mils, we need to know the angle and the height of a target to find the distance to it when using MOA. With mils the relationship of the arc height to the radius (or distance) was a nice ratio of some number "X" to 1,000 (see page 14 again). Here that is not the case. Therefore, we can use another method of math to come up with a distance equation.

In our drawing of the geometry of a shot, notice we have what looks like a "right triangle".



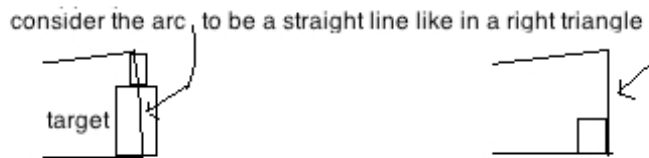
Recall, a right triangle has one angle of 90° (below):



The relationship of the sides and the angles of a right triangle are the basis of the subject of trigonometry. If you look at the picture above, the sides and the angle we're concerned with from a shooter's perspective are the opposite side, which can be thought of as the height of a target (we'll call that side "H"), the adjacent side, which for shooters, is the distance to the target (we'll call that side "D"), and the angle X, which will be in the units of MOA.

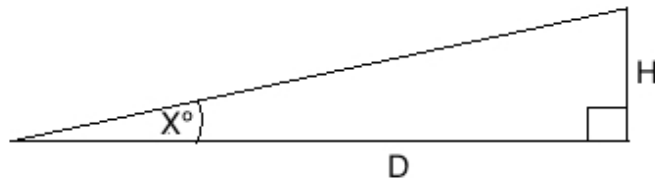
Note: We can disregard the hypotenuse for our discussion, as it does not factor into one of the variables we need to use to get a distance equation (picture above).

Just like in our discussion on mils, we can consider the opposite side, to be a straight line like it is on a triangle even though it is really an arc (see page 11 again). At the distances and angles shooters use, the effects of this on the math are negligible.



We will use the tangent function of trigonometry to figure out the unknown distance to a target with the height of the target and the angle known. The equation for that is:

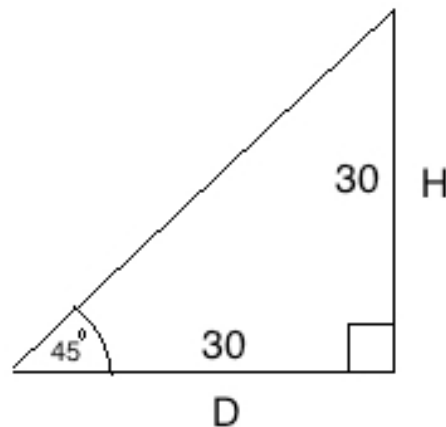
$$\tan(X) = \frac{H}{D}.$$



Note: If you'd like to skip the math, please jump to below the black line on page 29.

Let's do a quick review lesson on the Tangent function. What this Tangent function or equation says is that every angle of a right triangle (X in the previous picture) has a certain ratio of the opposite side H to the adjacent side D . The "tangent of an angle" is that ratio. Another way of saying that is, H divided by D will always be the same unique number for a specific angle of a right triangle, no matter how big the triangle is. Let me show you a simple example to help explain this.

With a 45° right triangle the sides H and D are always equal. That means the ratio of H to D is always going to equal 1 (below):



$$\tan 45^\circ = \frac{30}{30} = 1.$$

OR

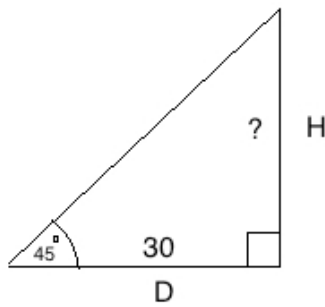
$$\tan 45^\circ = 1.$$

Therefore, the tangent of a 45° right triangle is **always 1**, no matter what the sides' lengths are, because the ratio of both sides, which are always equal

in length for a 45° right triangle, is "1". Now that we know this, let's look at this example from a different perspective.

Let's pretend we didn't know that a 45° right triangle has both sides that are equal in length. If we knew the angle was 45° and we only knew one side's length, how could we use the tangent equation to find out the other side's length?

Example: Pretend we know the length of side D is 30 but we don't know what the length of side H is. From above we know the ratio, or tangent, of a 45° angle is 1. Therefore, plug the numbers in the tangent equation and solve for the unknown side H :



$$\tan(45^\circ) = \frac{H}{30}.$$

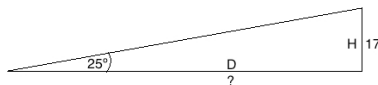
We know the $\tan(45^\circ) = 1$. Therefore,

$$1 = \frac{H}{30}$$

$$H = 30.$$

The Tangent of 45° is the nice round number "1", which makes for a very easy example to do. The tangents of all the other angles of a right triangle are not necessarily as nice though. In the old days you had to look up the tangents of angles in tables or charts to solve tangent equations, but today we can just use a calculator.

Here's another more realistic example on how to use the tangent function: In this example, the angle is (25°) , side H has a length of 17 and side D is unknown. Solve for " D ":



$$\tan(25^\circ) = \frac{17}{D}.$$

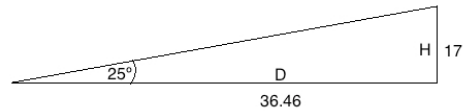
Using your calculator, the $\tan(25^\circ) = 0.4663$. Then,

$$0.4663 = \frac{17}{D}$$

$$0.4663D = 17$$

$$D = \frac{17}{0.4663}$$

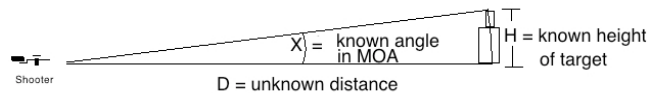
$$D = 36.46.$$



Using the Tangent function, we found the length of side D equals 36.46.

Now that we have reviewed how the tangent function works, we can now proceed to manipulating it so that we can come up with an equation for the distance to a target using MOA.

For shooters, as before with mils, the unknown value is the distance to the target, (or the adjacent side, D , in the triangle below). The known values are the angle in MOA and the height of the target. Setting up the tangent equation for this situation, we get:



$$\tan(X) = \frac{H}{D}.$$

Solving for D :

$$D \cdot \tan(X) = H$$

$$D = \frac{H}{\tan(X)}$$

The method I'm going to use to get a distance equation is based on the fact that **for small angles** the pie sector of the circle (see the pictures on page 23) is approximately a right triangle. If X represents an MOA, then these two equations are (essentially) equal:

$$D = \frac{H}{\tan(X)} = \frac{H}{X \cdot \tan(1 \text{ MOA})}.$$

What is being said in the equation above is instead of always looking up the tangent of a MOA value on your calculator as you have to in the equation on the left (above), all you have to do now is find out the value of the tangent of 1 MOA just once, put that number in the equation on the right, and let it sit there as a constant. This enables us to no longer have to look up the tangent every time when using the distance equation.

Let me show you what I mean with an example. Let the MOA angle, X , equal an MOA value of 4. Does: $\tan(4 \text{ MOA}) = 4 \tan(1 \text{ MOA})$?

Recall from page 22 that 1 MOA equals 1/60th of a degree. That means: $4 \text{ MOA} = 4/60\text{th}$ of a degree.

Now let's compare if: $\tan(4 \text{ MOA}) = 4 \cdot \tan(1 \text{ MOA})$.

$$\tan(4 \text{ MOA}) = \tan(4/60^\circ) = \tan(.066667) = \mathbf{0.0011635} \quad (1)$$

$$\text{Does } 0.0011635 = 4 \times \tan(1 \text{ MOA})? \quad (2)$$

$$\tan(1 \text{ MOA}) = \tan(1/60^\circ) = \tan(0.0166667) = 0.00029088 \quad (3)$$

$$4 \times 0.00029088 = \mathbf{0.0011635}. \quad (4)$$

Notice the bold numbers in lines (1) and (4) are equal. Thus, $\tan(4 \text{ MOA}) = 4 \cdot \tan(1 \text{ MOA})$.

This fact (**for small angles**) enables us in the equation below to calculate the tangent of 1 MOA and put that number in as a constant:

$$\frac{H}{X \cdot \tan(1 \text{ MOA})} = D.$$

From the example we just did above, we know the $\tan(1 \text{ MOA})$ is (.000290888). The equation now looks like this:

$$\frac{H}{X \cdot \tan(1 \text{ MOA})} = \frac{H}{X \cdot 0.000290888} = D.$$

That doesn't look much better, but at least we got rid of the step of calculating the tangent every time because we now have the constant

(0.000290888) in there instead. The good news is we can further simplify this equation. Let's convert (0.000290888) from a decimal to a fraction:

$$0.000290888 = \frac{2.90888}{10,000}$$

Now the equation can be written like this:

$$\frac{H}{X \cdot \tan(0.000290888)} = \frac{H}{X \cdot \frac{2.90888}{10,000}} = D.$$

We can further simplify it.

$$\frac{H}{X \cdot \frac{2.90888}{10,000}} = \frac{H \cdot 10,000}{X \cdot 2.90888} = \frac{H \cdot 3,437.75}{X} = D.$$

(→)

This is the distance equation in its simplest form for scopes using MOA as the unit of angular measurement. H is the height of the target and X is the angle in MOA:

$$\frac{H \times 3,437.75}{X} = D.$$

The equation still looks cumbersome though, and is not very user friendly. There is one more step we can do that will clean this up.

Recall from the section on mils that the distance equation gives you the distance to a target in the same units you used for measuring the height of a target (page 16). For American shooters, measuring the height of a target in inches and getting the distance to it in yards is advantageous. So let's make that substitution again here and clean up this equation.

Recall from page 17 that $H/36$ will convert inches to yards:

$$\frac{H \text{ in inches}}{36} = H \text{ in yards.}$$

That is, a height measured in inches divided by 36 equals the height in yards.

$$\text{Example: } \frac{36 \text{ inches}}{36} = 1 \text{ yard, } \frac{18 \text{ inches}}{36} = \frac{1}{2} \text{ yard.}$$

So let's put that conversion in the equation and simplify it:

$$\frac{H \text{ (yards)} \times 3,437.75}{X} = \frac{\frac{H}{36} \text{ (inches)} \times 3,437.75}{X} = \frac{H \times \frac{3,437.75}{36}}{X} = \frac{H \times 95.5}{X} = D.$$

With H measured in inches and the output D in yards, the MOA distance equation now is:

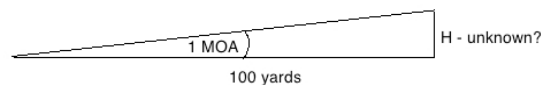
$$\frac{H \text{ inches} \times 95.5}{X \text{ MOA}} = D \text{ yards.}$$

This equation started out as the tangent equation (page 27), then turned into the basic equation (page 29) and finally into the one above. It's important to note that the basic equation can be adjusted for any units of measurement that you desire to use for the height of the target and the distance you want to get it in (below):

$$\frac{H \times 3,437.75}{X \text{ MOA}} = D.$$

The steps are the same like we just did with the "inches" to "yards" example (above). I won't go through the steps again, but you can see the results of that in the MOA equations on page 40.

Now that we have this equation, let's use it to show you where a familiar shooting expression comes from. In this example, we want to know how much 1 MOA equals in height at 100 yards. That is, we know the distance and we know the angle, but we want to see how high a 1 MOA angle is at 100 yards.



To do this I'm going to adjust the equation to solve for the unknown value of H .

$$\frac{H \text{ inches} \times 95.5}{X \text{ MOA}} = D \text{ yards.}$$

Solve for H by cross-multiplying:

$$H \text{ inches} = \frac{D \text{ yards} \times X \text{ MOA}}{95.5}.$$

Plug in the known numbers, 100 yards and 1 MOA.

$$H \text{ inches} = \frac{100 \text{ yards} \times 1 \text{ MOA}}{95.5}$$

$$H = 1.047 \text{ inches.}$$

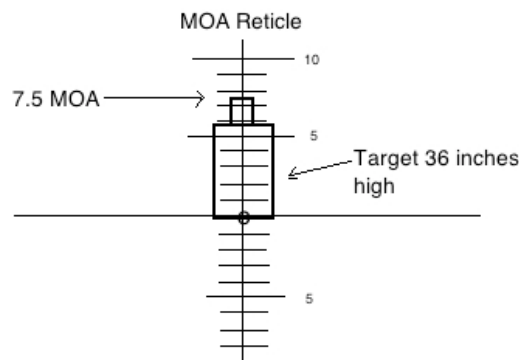
That is, at 100 yards, 1 MOA equals a height of 1.047 inches. Being so close to 1 inch, what many people commonly say though is:

“At 100 yards, 1 MOA equals 1 inch”.

It must pointed out though, it’s just a lucky coincidence that 1 MOA almost equals 1 inch at 100-yards, but it is very convenient.

Note: If you solve for the distance at where 1 MOA would give you exactly 1 inch, you would get 95.5 yards. That is, at 95.5 yards, 1 MOA equals 1-inch.

Here’s an example of how a shooter would use the MOA distance equation: A shooter sees a target that he knows is 36 inches high and it covers 7.5 MOA on his scope. How far away is it?



Plugging the numbers in the equation and calculating:

$$\frac{H \text{ inches} \times 95.5}{X \text{ MOA}} = D \text{ yards}$$

$$\frac{36 \text{ inches} \times 95.5}{7.5 \text{ MOA}} = 458 \text{ yards.}$$

The target is 458 yards away.

The MOA equation with the “inches to yards” conversion of 95.5 is very simple to use, and being that 95.5 is so close to 100, if you have to calculate quickly, you can round up and do the math in your head fairly accurately. This leads us to our next discussion.

6 Shooter Minute of Angle (S-MOA)

Inch per 100 yards (IPHY) or Shooters-MOA (S-MOA)

With 1 MOA being so close to 1 inch at 100 yards (1.047 inches at 100-yards), shooters and manufactures of scopes decided to use a slightly smaller angle that will cover exactly one inch at 100 yards, or “inch-per-hundred-yards” (IPHY). Being so close to the MOA value of 1.047 inches, easier to say than “inch-per-hundred-yards” and likely invented by shooters, the angle became more popularly known as “shooters” MOA (S-MOA).

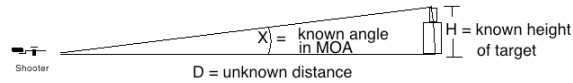
Note: Since “shooters MOA” is easier to say than “inch-per-hundred-yards”, I will mostly use that term from here on out.

Many scopes do have their reticles calibrated in S-MOA and/or have their adjustments in S-MOA instead of “true” MOA. This has advantages in ease of use and the range equation (as you will see). To find the angle that will give you exactly one inch at 100 yards, all we need to do is some simple trigonometry again.

S-MOA Range Estimation Equation

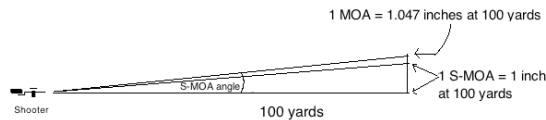
Note: If you’d like, you can skip the math and jump to below the black line on page 35.

Recall from page 27 the picture and the basic equation for finding the distance to a target:



$$D = \frac{H}{\tan(X)}$$

Our goal is to force the equation to give us the desired result of 1 inch high at 100-yards.



The way we can manipulate the equation to give us the desired result is to put the known values on one side of the equation and solve for the unknown value on the other side. In this case, the known values are a height of 1-inch and a distance of 100-yards (or keeping the units the same, 3,600 inches). The unknown value will be the angle. The question is, what angle will give us 1 inch at 3,600 inches (100 yards)? Let's use the tangent equation again and solve for the unknown angle:

$$\tan(X) = \frac{H}{D}$$

Plugging in the numbers:

$$\tan(X) = \frac{1 \text{ inch}}{3,600 \text{ inches}}$$

$1 \div 3,600 = 0.00027778$. Simplifying the equation we get:

$$\tan(X) = 0.00027778.$$

The equation above says that there is some angle out there that has the ratio (or tangent) of .00027778.

To get that angle we use what's called the "inverse tangent" function. Unlike the tangent function, which gives you the ratio of the opposite side to the adjacent side for a known angle (page 25), this function does the opposite of that; it gives you the angle if you are given a known ratio first. It usually looks like this on your calculator:

$$\tan^{-1}(X).$$

Therefore, taking the inverse tangent of (.0002777), you get the angle (.01591549°). This means: $\tan(.01591549^\circ) = (.00027778)$.

This new angle (.01591549°) will cover exactly 1 inch at 100 yards and as discussed above, it will be called "S-MOA". This angle is slightly smaller than the angle of 1 MOA, which recall is 1/60th (.01666667°) of a degree (page 22). This new angle (.01591549°) is actually 1/62.83 of a degree. The way to figure that out is to solve for the number that when divided into 1 will equal .01591549:

$$\begin{aligned}\frac{1}{X} &= 0.01591549 \\ \frac{1}{0.01591549} &= X \\ X &= 62.83.\end{aligned}$$

So we have that:

$$\frac{1}{62.83} = 0.01591549.$$

That also means that since there are 62.83 S-MOA per degree, and we have 360° in a circle, then there are:

$$\frac{62.83 \text{ S-MOA}}{1^\circ} \times 360^\circ = \frac{62.83 \text{ S-MOA} \times 360^\circ}{1^\circ} = 22,618.8 \text{ S-MOA in a circle.}$$

Since there are 22,618.8 S-MOA in a circle compared to 21,600 MOA in a circle, you can see that S-MOA is very close in size to MOA.

Recall back on page 28 when the statement was made that **for small angles**:

$$D = \frac{H}{\tan(X)} = \frac{H}{X \cdot \tan(1 \text{ MOA})}.$$

It's also true that if the angle, X, is an S-MOA angle, then:

$$D = \frac{H}{\tan(X)} = \frac{H}{X \cdot \tan(1 \text{ S-MOA})}.$$

Just like we did previously with MOA (page 28), we're going to simplify this equation to come up with a range equation where we don't have to figure out the tangent of an angle every time.

From the top of page 34 (this page), we know:

$$\text{The tangent of 1 S-MOA} = \tan(.01591549^\circ) = 0.00027778.$$

Let's convert that to a fraction, put it in the distance equation, and then simplify:

$$\begin{aligned}\frac{H}{X \cdot \tan(1 \text{ S-MOA})} &= D \\ \frac{H}{X \cdot 0.00027778} &= D \\ \frac{H}{X \cdot \frac{2.7778}{10,000}} &= D \\ \frac{H \cdot 10,000}{X \cdot 2.7778} &= D \\ \frac{H \cdot 3,600}{X} &= D.\end{aligned}$$

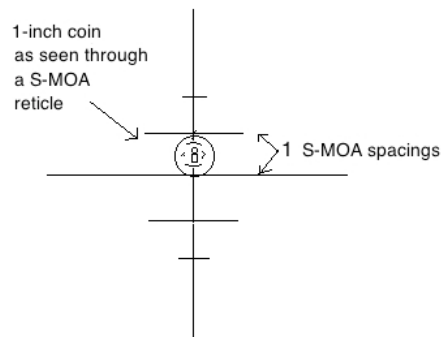
(→) The basic S-MOA equation is:

$$\frac{H \times 3,600}{X} = D.$$

The equation above says the height of an object in whatever units you use, multiplied by 3,600, then divided by the S-MOA angle, equals the distance to a target in the same units.

Originally we wanted an angle that would give us exactly 1 inch at 100 yards (which is the same as 1 inch at 3,600 inches) and here we have it. For example:

There is a target, a 1-inch diameter coin, and we see it fits exactly between 1 S-MOA on a scope. How far away is it?



$$\frac{1 \text{ inch} \times 3,600}{1 \text{ S-MOA}} = D$$

$$D = 3,600.$$

$D = 3,600$ inches, which as we know, is 100 yards. That is, a 1-inch object measuring 1 S-MOA on a scope is 100 yards away. Exactly what we originally wanted (pages 32 and 33).

Let's now put the "inches to yards" conversion that we have previously used (page 17) in the basic equation (previous page) to allow us to use inches for the height of an object yet get the distance to it in yards.

$$\frac{H \text{ (yards)} \times 3,600}{X} = \frac{\frac{H}{36} \text{ (inches)} \times 3,600}{X} = \frac{H \times \frac{3,600}{36}}{X} = \frac{H \times 100}{X} = D.$$

The new equation for S-MOA with the height measured in inches and the distance given in yards is:

$$\frac{H \text{ inches} \cdot 100}{X \text{ S-MOA}} = D \text{ yards.}$$

If you plug in a height of 1 inch at an angle of 1 S-MOA, the distance equals 100 yards, meeting the original goal of 1 inch at 100 yards.

Here's another more realistic example. You see a vehicle that has wheels you know are 28 inches in height and it covers 4 S-MOA on your scope. How far away is the vehicle?

$$\frac{28 \text{ inches} \cdot 100}{4 \text{ S-MOA}} = 700 \text{ yards.}$$

It can quickly and easily be calculated that it is 700 yards away.

7 Conversion Between Units

Recall back on page 21, when I said some scopes use mils for their reticle but had their adjustments in MOA, two different units of angular measurement. In that case, when you wanted to make adjustments to your shots using your elevation and windage controls, you first needed to convert the observed amount your shot is off target in mils to an equivalent MOA value for your elevation and windage adjustments.

For example, if you look through your scope and you observe that you are shooting 2.5-mils low at 750 yards, how much MOA up do you need to adjust your elevation control to correct for that?

Let's review. From pages 20 and 22 we know that we have 6,283 mils and 21,600 minutes in a circle. If you take 21,600 minutes and divide that by 6,283.2 mils, you get:

$$\frac{21,600 \text{ MOA}}{6,283 \text{ mils}} = 3.438 \text{ minutes per mil.}$$

That is, there are 3.438 MOA per every mil. This is the conversion between these two units of angular measurement. Therefore, in the example on the previous page, if you were 2.5 mils low at 750-yards, you need to adjust your scope:

$$2.5 \text{ mils} \times \frac{3.438 \text{ minutes}}{1 \text{ mil}} = 2.5 \text{ mils} \times \frac{3.438 \text{ minutes}}{1 \text{ mil}} = 8.6 \text{ minutes.}$$

You would need to adjust your scope 8.6 MOA up for the 2.5 mil low shot. If your scope had, as a lot do, 4 adjustment clicks per MOA in adjustment, then you would need to crank it up:

$$8.6 \text{ MOA} \times \frac{4 \text{ clicks}}{1 \text{ MOA}} = 8.6 \text{ MOA} \times \frac{4 \text{ clicks}}{1 \text{ MOA}} = 34.4 \text{ clicks.}$$

Since you can't dial up .4 clicks, you could either dial it up 34 or 35 clicks to make the adjustment. **Note however that the distance (700 yards) plays no part in the conversion between the units, and never does.**

In the example just given, you can see how the units of mils on the left cancel each other out so all you're left with is MOA on the right:

$$2.5 \text{ mils} \times \frac{3.438 \text{ minutes}}{1 \text{ mil}} = 8.6 \text{ minutes.}$$

Therefore, to make a correction from your mil reticle to your MOA adjustments, all you need to do is the following:

$$(\text{Correction in mils}) \times 3.438 = (\text{Correction in MOA}).$$

If you had a mil reticle and S-MOA adjustments, then the conversion is:

Recalling from page 34 that there are 22,618.8 S-MOA in a circle and we know there are 6,283 mils in a circle:

$$\frac{22,618.8 \text{ S-MOA}}{6,283 \text{ mils}} = 3.6 \text{ S-MOA per mil.}$$

That is, there are 3.6 S-MOA per every mil.

Note: That makes sense since we know that at 100 yards, 3600 inches, 1 mil equals 3.6 inches (page 8). Since we wanted our S-MOA angle to be exactly 1-inch at 3,600 inches, then there should be 3.6 of them per mil at 100-yards.

Therefore, the conversion between mils and S-MOA is:

$$(\text{Correction in mils}) \times 3.6 = (\text{Correction in S-MOA}).$$

It is very important to know what type of reticle you have and what your adjustments are calibrated in. Many scopes have mil reticles and MOA adjustments, but they could also have:

- Mil reticle and S-MOA adjustments
- Mil reticle and Mil adjustments
- MOA reticle and MOA adjustments
- S-MOA reticle and S-MOA adjustments

A major advantage of having your adjustments in the same units as your scope's reticle is that making adjustments to your shots is simple since there is no conversion required. All you have to do is measure how many mils, MOA or S-MOA (depending on whatever reticle you're using) your shot is off and adjust your elevation and windage controls the same number of units in the opposite direction, no matter the distance, with no conversion required. For instance, if you had a mil reticle and mil adjustment controls, then in the example on page 36 where your shot was 2.5 mils low, all you would need to do is adjust up 2.5 mils on your elevation control with no conversion required.

Conclusion

I originally wrote this paper in 2007-8 because even though I had scopes with different reticles in them, I really didn't understand how they worked. I was also curious about how they came up with the range estimation equations. Where did they come up with the numbers like 3437.75 or 95.5 used in some of the equations. There had to be some underlying basis for them and I wanted to find out what that was. Researching the subject, I couldn't find where the equations came from nor was I satisfied with the explanation of the angles. So I set out on a goal to find those answers myself, write them down in simple language, and pass that information along to other shooters. I hope I achieved that goal and I hope this knowledge makes you a better-educated and well-rounded marksman. Good shooting. Thank you.

Sincerely,

Robert J. Simeone

8 Distance Equations

Below are the distance equations for various combinations of reticles, units of height and distance. I derived them the same way I did on starting on page 16. Just put a conversion between units into the basic range equation and solve. Pick the equation that meets your needs in terms of your reticle and the units of measurement you prefer to use.

Mils

$$\frac{\text{Height of target (yards)} \times 1,000}{\text{mils}} = \text{Distance to target (yards)}.$$

$$\frac{\text{Height of target (inches)} \times 27.78}{\text{mils}} = \text{Distance to target (yards)}.$$

$$\frac{\text{Height of target (yards)} \times 25.4}{\text{mils}} = \text{Distance to target (meters)}.$$

$$\frac{\text{Height of target (meters)} \times 1,000}{\text{mils}} = \text{Distance to target (meters)}.$$

$$\frac{\text{Height of target (cm)} \times 10}{\text{mils}} = \text{Distance to target (meters)}.$$

MOA

$$\frac{\text{Height of target (inches)} \times 95.5}{\text{MOA}} = \text{Distance to target (yards)}.$$

$$\frac{\text{Height of target (inches)} \times 87.32}{\text{MOA}} = \text{Distance to target (meters)}.$$

$$\frac{\text{Height of target (meters)} \times 3,437.75}{\text{MOA}} = \text{Distance to target (meters)}.$$

$$\frac{\text{Height of target (cm)} \times 34.37}{\text{MOA}} = \text{Distance to target (meters)}.$$

S-MOA

$$\frac{\text{Height of target (inches)} \times 100}{\text{S-MOA}} = \text{Distance to target (yards)}.$$

$$\frac{\text{Height of target (inches)} \times 91.44}{\text{S-MOA}} = \text{Distance to target (meters)}.$$

$$\frac{\text{Height of target (meters)} \times 3,600}{\text{S-MOA}} = \text{Distance to target (meters)}.$$

$$\frac{\text{Height of target (cm)} \times 36}{\text{S-MOA}} = \text{Distance to target (meters)}.$$

9 Quick Reference Guide

- At 1,000 yards, 1 mil equals 1 yard: page 6.
- At 1,000 meters, 1 mil equals 1 meter: page 7.
- At 100 yards, 1 mil equals 3.6 inches: page 8.
- At 100 meters, 1 mil equals 10 cm: pages 8 and 9.
- Basic Distance equation for mils: page 16

$$\frac{H \times 1,000}{X \text{ mils}} = D.$$

- Common mil distance equation for American shooters: page 18.

$$\frac{H \text{ inches} \times 27.8}{X \text{ mils}} = D \text{ yards.}$$

- Basic MOA equation: page 29.

$$\frac{H \times 3,437.75}{X \text{ MOA}} = D.$$

- Common MOA distance equation for American shooters: page 30.

$$\frac{H \text{ inches} \times 95.5}{X \text{ MOA}} = D \text{ yards.}$$

- 1.047 inches per “true” MOA at 100 yards: page 31.
- Basic Shooters MOA (S-MOA) distance equation: page 35.

$$\frac{H \times 3,600}{X \text{ S-MOA}} = D.$$

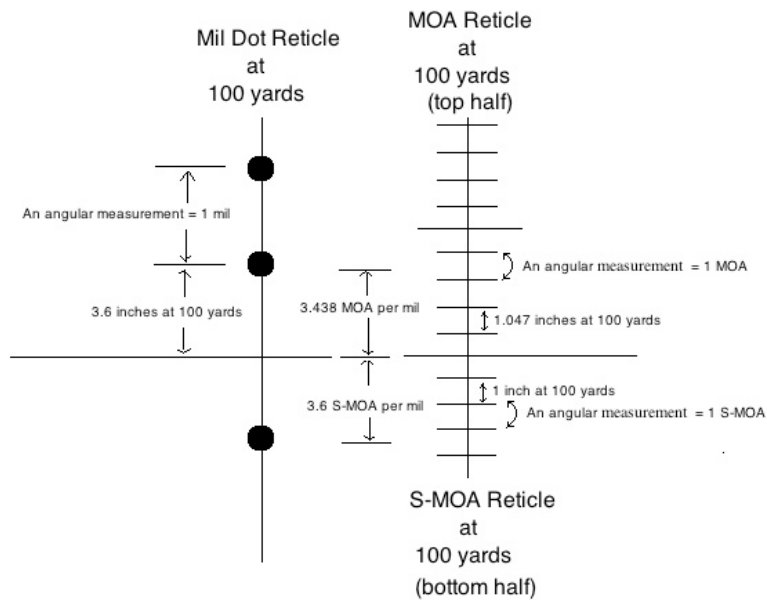
- 1 inch per “shooters” MOA (S-MOA) at 100 yards: page 36.
- Common MOA distance equation for American shooters: page 36.

$$\frac{H \text{ inches} \cdot 100}{X \text{ S-MOA}} = D \text{ yards.}$$

- 1 mil = 3.438 minutes (MOA): page 37.
- (Correction in mils) \times 3.438 = (Correction in MOA): page 37.
- 1 mil = 3.6 S-MOA: page 37.
- (Correction in mils) \times 3.6 = (Correction in S-MOA): page 38.

10 Comparison of Angles

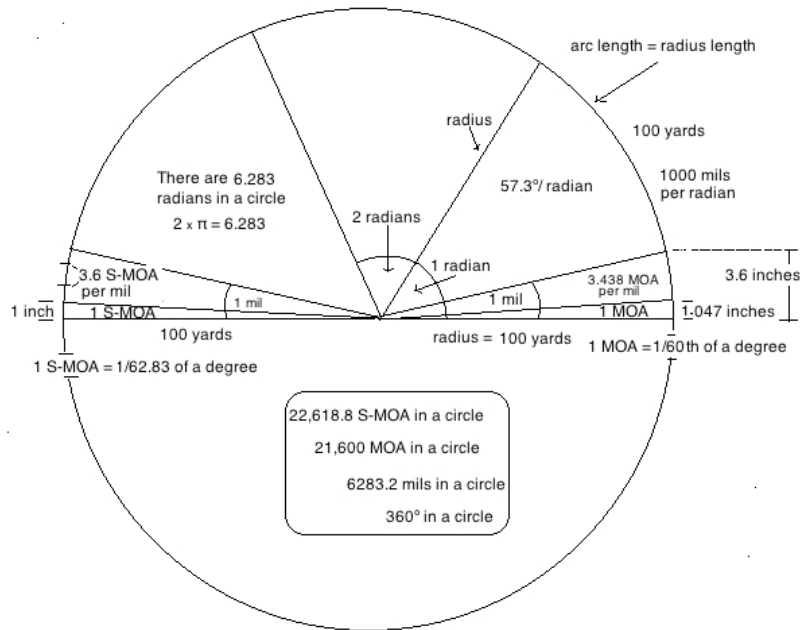
This is a comparison of the three angular measurements (mils, MOA and S-MOA) at 100 yards. Don't confuse the angular measurement with what they cover in height. For example, at 100-yards, 1 mil covers 3.6 inches in height but is equal to the "angular measurement" of 3.438 MOA.



Note: Drawing not to scale.

11 The Big Picture

The Big Picture at 100 yards
(Note: The figure below is not to scale).



12 Acknowledgements

I would like to thank my daughter, Shauna Rae, for reviewing my math, and for editing and reformatting this paper.

Additionally, I would like to thank Peter Froncek for also using his math expertise to edit this paper.