Tuning the Rifle Barrel and Load Together

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1. Why Tune for Bullet Launch with Zero Yaw?

If the rifle bullet is given a non-zero aeroballistic yaw and yaw-rate as it clears the muzzle, those attitude errors can only be amplified in their magnitudes by passage of the bullet through the muzzle-blast zone prior to commencing its ballistic flight. These amplified attitude errors are quite variable from one shot to the next. Even when firing through a wind-free atmosphere, any initial aeroballistic yaw results in an aerodynamic jump deflection of the entire remaining ballistic trajectory and in increased yaw-drag during the critical highest-drag beginning portion of the bullet's ballistic flight. This higher initial air-drag then causes increased sensitivity to any ambient crosswinds immediately in front of the firing point. Another odd effect of non-zero initial yaw which has been observed is a different response to crosswinds of opposite directions depending upon the sense of the rifling twist (right-hand or left-hand twist). Non-zero initial aeroballistic yaw-rate is an even worse offender than initial yaw itself in each of these aspects.

Because the CG of the typical rifle bullet is well forward of its point of last contact with the muzzle during its exit process, the bullet can exit without acquiring any non-zero yaw and yaw-rate only when the muzzle is stationary in inertial space during bullet exit. If the muzzle of the rifle barrel is in motion during bullet exit, it will tip the nose of the fired bullet in the direction opposite to that of the muzzle motion and impart a yaw-rate which works to increase that non-zero attitude of the bullet's spin-axis orientation relative to its forward velocity direction. Subsequent passage of the bullet through the muzzle-blast zone amplifies both the mechanically imparted yaw and yaw-rate errors by significantly variable amounts. The result of these initial attitude errors in ballistic flight are random spreads in target impact points and random variations in effective Ballistic Coefficient (BC).

2. Recoil Effects on Rifle Barrel

Rifle recoil occurs only in reaction to the forward acceleration of the rifle bullet during the firing of the rifle. Thus, the line-of-action of this rearward recoil force is coaxial with the bore of the rifle barrel. This recoil force acting on the rifle can be quantified at any time during the firing process as the base-pressure driving the bullet forward multiplied by the cross-sectional area of the bore being obturated by that bullet. After initial engraving of the rifle bullet by the rifling lands, the forward-acting force of barrel friction is less than 2-percent of the bullet’s driving force and is considered negligible here. This rearward recoil force on the rifle can be thought of as being applied to the rifle at the center of its breech face as a portion of the thrust of the cartridge case head against it plus whatever rearward frictional force might obtain between the case walls and the chamber walls.

The CG of most shoulder-fired rifles being fired upright is located several millimeters (dCG) vertically below the axis of its bore. Thus, the recoil force briefly creates a vertically upward bending torque at the front face of the receiver acting upon the rear of the (assumed) free-floating rifle barrel. Here, we are analytically treating this upward torque as being impulsive, occurring at the instant of peak base-pressure driving the bullet forward in the rifle barrel. This treatment allows the rather precise timing of the barrel’s dynamic response at its muzzle to this torque impulse being applied at its receiver end. We are not concerned here with the muzzle of the barrel being dragged straight rearward during recoil. Good rifle design and firing technique should eliminate any disturbance of the rifle barrel in the horizontal plane during firing.

The recoil torque impulse imparts a transverse shear-wave in a vertical plane into the material of the rifle barrel at its junction with the receiver face. The driving recoil torque actually looks like the first half of a sine wave having a frequency which we term the “peak excitation frequency.” More closely inspecting the base-pressure curve plotted against time in a good interior ballistics application such as QuickLOAD©, we see that it appears distinctly Gaussian in its excitation profile, having a “standard deviation,” **sigma(seconds)**, given by its rise time from 60.65-percent to 100-percent of peak base-pressure. The excitation spectrum in the frequency domain is given by the Fourier transform of this time-domain base-pressure curve. We know that this excitation spectrum must then also be a Gaussian function of frequency centered at the peak excitation frequency mentioned earlier and having a spread function **sigma(hertz)** inversely related to that of its Gaussian time-domain driving function **sigma(seconds)**:

$$\textbf{Sigma(hertz)} = 1/[(\pi^2)*\textbf{Sigma(seconds)}]$$

If we model the rifle barrel mechanically as a “long, slender rod” of an isotropic material (steel) having a uniform cross-section, we can use engineering handbook data to calculate its response to this forced shear-wave initial distortion. Specifically, the rifle barrel is modelled as a uniform cantilever beam having a clamped end at its receiver junction and a free end at its muzzle.

The interior ballistics program gives us the time of peak base-pressure and the time of bullet exit from the muzzle of our rifle barrel. We can calculate rather precisely the “signaling delay” between the time the shear-wave is introduced at the receiver/barrel junction and the earliest time when the muzzle of the barrel begins to react to this recoil-driven torque impulse:

Signaling Delay = Barrel Length/Shear-Wave Propagation Speed

Here we need the input Barrel Length (**L”**) as measured externally from receiver face to muzzle. We calculate the shear-wave propagation rate along the barrel as a “long, slender rod” from the properties of the steel barrel material. This speed is about **3050 meters per second** for 416R stainless steel rifle barrels.

The muzzle begins to vibrate sinusoidally in a transverse vertical plane as soon as the upward bending torque (shear-wave) signal reaches it. These vibrations are of multiple modes, all starting simultaneously, with each mode continuing at its own specific mode frequency which we calculate from handbook data for the barrel as a cantilever beam. The actual muzzle motion is then the algebraic sum of the sinusoidal vibration modes at their respective excitation amplitudes evaluated at the muzzle.

We seek to tune the muzzle exit times of our fired bullets to match a reversal time (halt) in vertical muzzle motion when the muzzle is momentarily stationary relative to the earth as a quasi-inertial space. We can achieve partial “compensation” for long-range gravity-drop variations with variations in bullet launch velocities by tuning our mean bullet exit times to occur just earlier than an upward-to-downward muzzle halt, or just after a downward-to-upward muzzle reversal. This partial gravity-drop compensation is based upon a likely strong inverse correlation between variations in bullet launch velocities and variations in muzzle exit times for groups of shots fired sequentially.

3. Barrel Transverse Vibration Modes

For a uniform cantilever beam, its natural transverse shear-wave vibration **Mode(n)** frequencies are determined primarily by its beam length **L** and secondarily by its flexural rigidity **E*I** and its mass per unit length **A*ρ**, where **L** is the barrel’s “vibrational length,” **E** is Young’s Modulus of Elasticity for the barrel steel, **I i**s the second moment of cross-sectional area (**A**), and **ρ** is the density (mass per unit volume) of the barrel steel.

For a rifle barrel of uniform outside diameter **D** and caliber **d**, the second moment of area **I** is given by

$$I = (\pi/32)*(D^4 - d^4)$$

and the uniform cross-sectional area **A** is given by

$$A = (\pi/4)*(D^2 - d^2)$$

The attached **Sheet 1** shows an image of a ***Data Input*** spreadsheet complete with units conversion utilities to allow inputs in British Engineering units to be converted conveniently into metric SI (MKS) units for these calculations.

The transverse vibration modes are numbered according to the count (**n**) of the vibration nodes (locations of zero vibrational amplitude) occurring over the beam length L for that vibrational mode shape. The (clamped) barrel/receiver joint is always considered to be a vibration node, while the (free) muzzle end is always a vibrational anti-node. Each segment of rifle barrel material vibrates independently at each mode frequency in a sinusoidal motion with amplitudes constrained within the mode shape envelopes. In particular, the muzzle end segment of the barrel vibrates transversely in a vertical plane according to the instantaneous sum of all of the first seven, or so, sinusoidal vibration modes, so that the muzzle vertical position **y(t)** is given at any time **t** after excitation time t_0 by:

$$y(t) = \Sigma_{(n=1,\,7)}\{A(n)*SIN[2*\pi*f(n)*(t - t_0)]\}$$

The mode frequencies **f(n)** are given in hertz by:

$$f(n) = [1/(2*\pi)]*[a(n)/L^2]*SQRT[E*I/(A*\rho)]$$

The mode **n** frequency constants **a(n)** are taken from the handbook by Blevins, *Formulas for Natural Frequencies and Mode Shapes*, 1979, for a cantilever beam with clamped and free ends.

The mode frequency calculations are shown for a worked example in the attached **Sheet 2**. The mode vibration Amplitudes **A(n)** are found by multiplying an overall excitation amplitude by the relative amplitudes from the excitation spectrum for each mode frequency. The overall excitation amplitude is calculated using basic physics, and the Relative Excitation Amplitudes **REA(n)** are found from the Gaussian function:

$$REA(n) = EXP\{-0.5*[(f(n) - f(peak))/sigma(hz)]^2\}$$

The mode shapes are also shown graphically in **Sheet 2** of the worked example. Note that with an upward driving torque applied at the receiver end of the barrel, the muzzle end is initially moved *upward* by each odd-numbered vibration mode and *downward* by each even-numbered vibration mode.

4. Barrel Taper and Muzzle Attachments

The calculations of muzzle position as a function of ongoing time **y(t)** are quite accurate for rifle barrels of constant outside diameter **D** and having no muzzle attachment. We have formulated simple extensions of these calculations to handle those rifles having conventionally tapered barrels and/or lightweight muzzle attachments producing approximate results which should be good enough to be in the ballpark. Ultimately, there is no substitute for test firing in ambient shooting conditions for rifle tuning.

As shown in **Sheet 1**, we accept input of barrel mass, if that is accurately known, and calculate an Average Diameter to use for tapered barrels. Alternatively, we can accept a Midpoint barrel OD measurement and estimate barrel mass from that. We also accept an unthreaded muzzle OD measurement and calculate a Vibrationally Effective Diameter to use in vibration calculations as its geometric mean with the calculated Average Diameter, which biases toward the smaller muzzle OD.

Without requiring more details, we simply accept the mass of any muzzle attachment and calculate an adjusted Vibrational Length of the rifle barrel by extending its rod-measured internal length at its Average Diameter to match the total mass of the barrel and attachment. This approach seems to work well enough for muzzle brakes weighing a few ounces, but is speculative for heavier suppressors, for example. The vibrational coupling with a heavy suppressor attached to the muzzle via a single threaded joint is questionable in any case.

5. Worked Example

The attached images show screenshots of a worked example using a “live” Excel workbook of four spreadsheets which is freely available upon request from the author as an email attachment. In addition to having access to a current version of Microsoft Excel, the user will also need access to a good interior ballistics program such as QuickLOAD©, which we routinely use in load development here. In particular, this analysis uses the QL definition of start time (**t = 0**) as the time of chamber pressure reaching 10-percent of its peak value (**P-Max**). Other interior ballistics programs might use time-since-sear-break, or time-since primer ignition, either of which would occur much earlier, and their event timings would require adjustment for use here. [Just note the time of **10% P-Max** and subtract it from all other event times.]

The example shown is for a known accurate load in a heavy-barrel 6.5x47 Lapua rifle using a 27-inch barrel of constant 1.250-inch OD and having no muzzle attachment. The bullet is a 153-grain Hornady A-Tip propelled by 39.2-grains of the new Alliant ReLoader-16 temperature compensated powder incorporating an anti-copper-fouling additive.

Sheet 1 shows the Data Inputs, **Sheet 2** shows the calculation of the Mode Frequencies and Shapes, **Sheet 3** shows the relative Excitation Spectrum, and **Sheet 4** shows the Calculated Results, including a graph of muzzle position **y(t)** in fractions of a millimeter versus time **t** in microseconds. The table of Event Times shows the QL-calculated bullet exit occurring at **2 microseconds** after the muzzle has reversed its downward movement and is starting back upward again. This timing match is excellent for launching these match-type bullets with zero initial yaw and yaw-rate while also producing somewhat "compensated" long-range accuracy with this bullet and powder load. The calculated vertical muzzle speed is upward (positive) at a mere **0.045 mm/sec**.

Do not peremptorily dismiss these tiny calculated vibrational amplitudes and muzzle speeds as being so small that they can safely be ignored. They are perfectly capable of ruining the expected performance of good rifles and ammunition.

You will usually find that the peak excitation frequency (here **486.38 hz**) falls between the natural **Mode 2** frequency (here **434.58 hz**) and **Mode 3** frequency (here **1216.82 hz**). In this example, **Mode 2** is the dominant muzzle vibration with the faster **Mode 3** modulation providing the critical reversals in vertical direction muzzle motions. "Your results will vary," as is often said.

6. Summary

If you find that you lack sufficient control authority in tuning your bullet weight and propellant load choices for bullet exit at or very near one of the plotted reversal times, your choice of rifle barrel length and weight simply cannot fire bullets of your selected caliber and chambering optimally. Shortening an existing long barrel can usually allow proper load tuning with the desired bullet and powder. Using this analytical tool during rifle design can avoid making that costly mistake. If you have never fired a properly tuned load in one of your rifles, you are in for an eye-opening treat. "Benchrest accuracy" truly can be achieved by all riflemen.

Sheet 1. Data Inputs

Data Input [With Units Conversions to MKS Units]:

[Sample data values for a known accurate Heavy Barrel 6.5x47 Lapua are included and must be modified as needed for your rifle.]
[MKS Data Values to be used in subsequent calculations are all in **Column F**. Most have to be copy/pasted or manually entered.]
[Formulations used are noted in comments.]
[User Inputs for units conversion are indicated in **Bold**.]

Barrel Material Specification (isotropic steel)			Steel Type =	416R				
Density (rho) =				7750.00	kg/m^3			
Units Conversions:	lbm/ft^3	Kg/m^3						
	484.000	7752.94						
	483.817	**7750.00**						
Young's (Linear) Modulus of Elasticity (E) =				200.00	GPa			
Shear Modulus of Elasticity (G) =				72.30	GPa			
Units Conversions:	Pounds/Sq. In. (psi)	GigaPascals						
	29,000,000	199.95						
	29,007,368	**200.00**						
Transverse Shear-Wave Propagation Rate =			SQRT(G/rho) =	3054.35	m/s	120,250	in/sec	
Bore Inside Diameter (Caliber, d) =				0.006706	meters	0.2640	inches	
Units Conversions:	Inches	meters						
	0.2640	0.006706						
	0.2640	**0.006706**						
Actual Barrel Length (L') from Bolt-Face to Muzzle =				0.685800	meters	[Measured with a rod]		
Units Conversions:	**27.0000**	0.685800						
Actual Barrel Length (L") from Receiver-Face to Muzzle =				0.660400	meters	[Measured Externally]		
Units Conversions:	**26.0000**	0.660400						
Barrel Mass (M' if known) Excluding Any Muzzle Attachment (kg) =								
Pounds =	**8.8632**	4.0203						
Ounces =	**0.0000**	0.0000						
			Sum (kg) =	4.0203	kg	or	8.8632	pounds
Estimated Barrel Mass (M') based on M' = rho*L"*(Pi/4)*(Dm^2 - d^2):								
D(Midpoint) = Dm =	**1.2500**	0.031750		4.0203	kg	or	8.8632	pounds
Muzzle Attachment Mass =								
Pounds =	**0.000**	0.0000						
Ounces =	**0.000**	0.0000						
			Sum (kg) =	0.0000	kg	or	0.0000	pounds
Total Barrel Mass (M) Including Any Muzzle Attachment =				4.0203	kg	or	8.8632	pounds
Unthreaded Muzzle OD of Barrel (D") =				0.031750	meters	or	1.2500	inches
Average OD of Barrel (D') =				0.031750	meters	or	1.2500	inches
Vibrationally Effective OD of Barrel (D) = SQRT(D" * D') =				0.031750	meters	or	1.2500	inches
Vibrationally Effective Barrel Length with Attachment (L) = [Extending the Average OD (D') to Match Total Mass (M)]				0.685800	meters	or	27.0000	inches
Measured Rifle CG Offset (dCG) Below Bore Axis = [Measured at CG by Suspending Rifle via Bore vs Plumb Line]				0.010000	meters	or	0.3937	inches

Suggested Steel Properties:

		Cro-Mo	Stainless	
Steel Type =		4140	416R	
Density	rho =	7850	7750	kg/m^3
Linear Elasticity	E =	205.00	200.00	GPa
Bulk Elasticity	B =	160.00	160.00	GPa
Shear Elasticity	G =	80.00	72.30	GPa
Poisson's Ratio	nu =	0.29	0.30	
Linear Rod "Speed of Sound" = SQRT(E/rho) =		5110.25	5080.01	m/s
Bulk Material "Speed of Sound" = SQRT(B/rho) =		4514.66	4543.69	m/s
Transverse Shear-Wave Speed = SQRT(G/rho) =		3192.35	3054.35	m/s

Interior Ballistics Data [All times in microseconds (mu-sec) based on t = 0 at 10-percent P-Max]

Rise time to 60-percent Peak Base-Pressure (t60) =		341	mu-sec	
Time of Peak Base Pressure (tp) =		514	mu-sec	[About 0.98% of Time to P-Max]
Time of Bullet Exit from Muzzle (tb) =		1328	mu-sec	[Affected by Barrel Length]
Bullet Muzzle Velocity (fps) =	**2754**	839.42	m/s	[Affected by Barrel Length]
Peak Base Pressure (CIP Transducer psi) =	**54,140**	373.28	MPa	[About 90% of P-Max]

Sheet 2. Natural Mode Frequencies and Shapes

Barrel Transverse Shear-Wave Natural Mode Frequencies (Calculated in MKS Units)

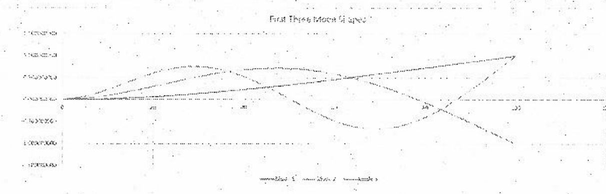
[Clamped receiver end of barrel is always a transverse vibration node.]
[Transverse Mode Number = Number of Nodes.]

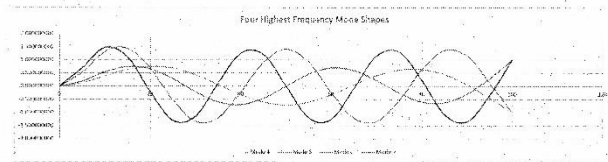
Barrel Inputs:		MKS Units		
Vibrational Length (L)		0.685603	meters	
Caliber (d)		0.006706	meters	
Total Barrel Mass (lbm, Kg)		4.020	kg	
Effective Diameter (D)		0.031780	meters	
416R Stainless Steel:				
Elasticity (E)		200.00	Giga Pascals (GN/m^2)	N = kg*m/sec^2
Density (rho)		7750.00	kg/m^3	
Transverse Wave Speed		3054.35	m/sec	

Calculated Values:			
Areal 2nd Mom (I) =	(Pi/32)*[D^4 - d^4]	9.95655E-08	meters^4
Cross-Section Area (A) =	(Pi/4)*[D^2 - d^2]	0.000756411	meters^2
Transverse Shear-Wave Speed	SQRT[G/rho] =	3054.35	meters/second
Flexural rigidity	E*I =	19913.10	kg*m^3*sec^-2
	I/A =	0.000131629	meters^2
Mass per unit length	A*rho =	5.8622	kg/meter
	SQRT[E*I/(A*rho)] =	58.2827	m^2/second
Natural frequency constant	SQRT[E*I/(A*rho)]/L^2 =	123.9201	radians/second

Cantilever Beam Mode Frequency Eqn: omega(n) = 2*Pi*f(n) = [a(n)/L^2]*SQRT[E*I/(A*Rho)]
[Clamped-Free Ends]

First 7 Transverse Modes: Mode No. (n)	[a(n) = Lambda(n) Squared] a(n) (Blevins)	Mode Freqencies omega(n) (radians/sec)	Mode Freqencies f(n) (hertz)
1	3.516015	435.70	69.34
2	22.034492	2730.52	434.58
3	61.697214	7645.52	1216.82
4	120.901916	14982.17	2384.49
5	199.859530	24766.60	3941.73
6	298.555533	36997.02	5888.26
7	416.990786	51673.52	8224.10
(etc.)	[(Pi/2)*(2*n - 1)]^2		





Barrel Natural Mode Shapes Normalized for Unit Amplitude at Muzzle

[Amplitude Envelope of Sinusoidal Vibrations for each Location along Barrel Length (L)]
[Let X = Percent of Barrel Length (L) measured from Receiver Face to Muzzle.]

[After Blevins, *Formulas for Natural Frequency and Mode Shape*, 1979]
[For Cantilever Beam with Clamped and Free Ends]

Lambda(n) =	1.87510407	4.69409113	7.85475744	10.99554073	14.13716839	17.27875960	20.42035225
Sigma(n) =	0.734095514	1.018467319	0.999224497	1.000033553	0.999998550	1.000000000	1.000000000
X (percent L)	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7
0	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000
5	0.004294191	0.025348713	0.067037978	0.123466349	0.191118761	0.532456873	0.690369781
10	0.016773500	0.092629298	0.228068902	0.385008085	0.537246470	1.321784175	1.474761366
15	0.036833444	0.188785060	0.424800029	0.633749798	0.747563060	1.450022610	1.122121202
20	0.063870931	0.301054890	0.604505968	0.753790258	0.659624791	0.673603083	-0.204392958
25	0.097285808	0.417259094	0.724499883	0.685159226	0.285200206	-0.527892093	-1.300497160
30	0.136482937	0.526132811	0.756239363	0.433869530	-0.211286962	-1.339389399	-1.141937731
35	0.180674771	0.617674438	0.686088113	0.065739434	-0.599307290	-1.203451675	0.111745043
40	0.229884375	0.683469448	0.525924626	-0.315569539	-0.696546857	-0.220235461	1.260357056
45	0.282948851	0.718965009	0.291744020	-0.600551335	-0.458155657	0.918878162	1.205917424
50	0.339523113	0.713665832	0.019687588	-0.707118629	0.000851469	1.414390559	0.000036793
55	0.399083963	0.671233632	-0.249292938	-0.605285383	0.460302042	0.918532839	-1.205802031
60	0.461134554	0.589475938	-0.473765256	-0.326494931	0.700254532	-0.221200301	-1.260068735
65	0.525208787	0.470217237	-0.618051292	0.045213331	0.606507222	-1.205802033	-0.110966196
70	0.590876299	0.317061799	-0.657425954	0.397390753	0.225728893	-1.344991436	1.144123426
75	0.657747305	0.134983613	-0.581451671	0.621421359	-0.255994882	-0.541193740	1.308563168
80	0.725477692	-0.070035863	-0.394873741	0.643038086	-0.600445869	0.642040520	0.221231820
85	0.793774179	-0.291657173	-0.115831701	0.441656182	-0.627580010	1.375139149	-1.075376403
90	0.862399544	-0.523751761	0.226507453	0.052034576	-0.293997027	1.144122978	-1.344997004
95	0.931177901	-0.761081476	0.608479988	-0.453590555	0.302093996	0.110957985	-0.330141604
100	1.000000001	-1.000000004	0.999999691	-0.999996244	1.000037467	-0.999999976	1.000000000

Sheet 3. Excitation Spectrum

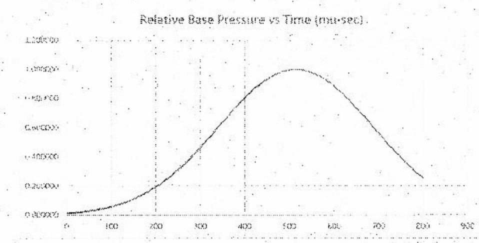
Excitation Spectrum [Modeling Base-pressure Curve as a Gaussian Function]

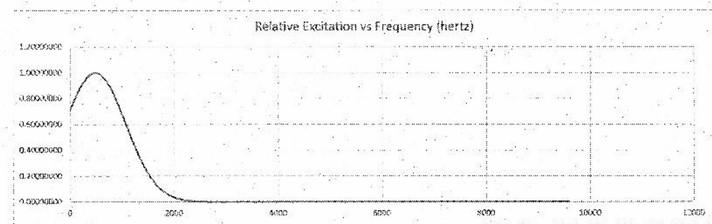
Time to Peak Base Pressure (mu-sec) =		514	microseconds
Time to 60% P-Max (mu-sec) =		341	microseconds
Sigma (mu-sec) =	[delta t] =	173	microseconds

Frequency Domain Function is Fourier Transform of Time Domain Function

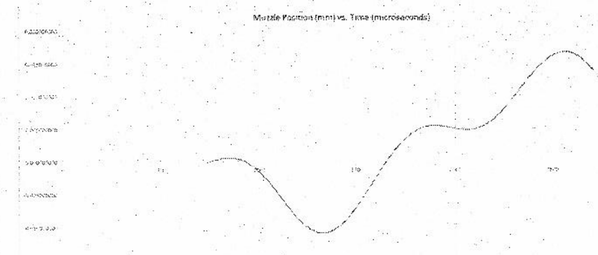
Peak Excitation Frequency (hertz) =	1000000/(4*tb) =	486.38	hertz
Sigma (hertz) =	1/[Sigma(time)*(Pi)^2] =	586	hertz

Time (mu-sec)	Rel. Base Pressure	Frequency (Hz)	Relative Excitation
0	0.012110	0	0.70833489
20	0.016960	50	0.75761213
40	0.023436	100	0.80443302
60	0.031955	150	0.84794475
80	0.042993	200	0.88731927
100	0.057076	250	0.92177933
120	0.074765	300	0.95062385
140	0.096637	350	0.97326163
160	0.123248	400	0.98918213
180	0.155101	450	0.99807248
200	0.192595	500	0.99972968
220	0.235978	550	0.99411766
240	0.285294	600	0.98135848
260	0.340338	650	0.96172800
280	0.400611	700	0.93564594
300	0.465298	750	0.90366092
320	0.533265	800	0.86643136
340	0.603025	850	0.82470288
360	0.672869	900	0.77928363
380	0.740836	950	0.73101836
400	0.804838	1000	0.68076263
420	0.862762	1050	0.62935810
440	0.912577	1100	0.57760991
460	0.952452	1150	0.52626700
480	0.980873	1200	0.47600589
500	0.996731	1250	0.42741840
520	0.999399	1300	0.38100334
540	0.988770	1350	0.33716233
560	0.965267	1400	0.29619929
580	0.929813	1450	0.25832335
600	0.883769	1500	0.22365469
620	0.828854	1550	0.19223260
640	0.767031	1600	0.16402527
660	0.700396	1650	0.13894060
680	0.631059	1700	0.11683751
700	0.561037	1750	0.09753716
720	0.492163	1800	0.08083372
740	0.426013	1850	0.06650431
760	0.363858	1900	0.05431773
780	0.306645	1950	0.04404212
800	0.254998	2000	0.03545108
		2050	0.02832862
		2100	0.02247274
		2150	0.01769789
		2200	0.01383635
		2250	0.01073881
		2300	0.00827419
		2350	0.00632892





Sheet 4. Results



Muzzle Position vs Time Since 10% of P-Max

[All Vibrations Initiate at Time of Peak Base-Pressure plus Signalling Delay.]
[Odd Modes Start Positive Going, and Even Modes Start Negative Going.]

Peak Torque (Gamma) at Rcvr. End =	131.8422	Kg m^2/sec^2
Torque-Driven Muzzle Amplitude (A) =	8.6870	micro-meters

Muzzle Motion Physics:
y = Muzzle Vertical Position
y(max) = (1/2)*y-double-dot*(2*delta t)^2
delta t = t(Peak Base-Pressure) - t(60% Peak)
Gamma = (dCG)*Base-Pmax*(pi/4)*d^2 = Iy(end)*omega-dot
Iy(end) = (1/3)*M*L^2
omega-dot(max) = (y-double-dot)/L = [2*y(max)]/[L*(2*delta t)^2]
y(max) = (3/2)*[(2*Delta t)^2]*[Gamma/(M*L)]

A(n) = A*Rel. Exc.(n)

Rel. Exc.(Mode n) = EXP[-0.5*((f(n) - Peak Freq.)/sigma(hz))^2]

Input Data	microsec		millimeters	Natural Modes	hertz	rad/sec	Relative Excitation	Table Look-ups: Time (Mu-sec)	Table Index	Value	
				Sigma(freq, hz) =	586						
				Peak Excit. Freq. =	486.38	3056.02					
Peak Base Pressure	514	A1 =	0.00666405	f1 =	69.34	435.70	0.77606507	1327	597	-0.01074744	mm
Signalling Delay	216	A2 =	0.00866345	f2 =	434.58	2730.52	0.99609539	1328	598	-0.010747545	mm
Start of Muzzle Motions	730	A3 =	0.00394524	f3 =	1216.82	7645.52	0.45944485	1329	599	-0.010747353	mm
Peak Excitation Freq (hz)	486.38	A4 =	0.00004498	f4 =	2364.49	14882.17	0.00523845	MAX/MIN	Muzzle speed =	0.0439	mm/sec
Bullet Exit Time (mu-sec)	1328	A5 =	0.00000000	f5 =	3941.73	24766.60	0.00000003	846	116	0.008886194	mm
Shear Propagation Rate (m/s	3054	A6 =	0.00000000	f6 =	5888.26	36997.02	0.00000000	1326	596	-0.010747645	mm
Barrel Length (L")	0.6604	A7 =	0.00000000	f7 =	8224.10	51673.52	0.00000000				

Event Times	(microsec)			
1. Chamber Press 10% of P-Max	0			
2. Peak Base Pressure	514			
3. Start of Muzzle Motions	730			
4. Upward Halt	846			
5. Bullet Exit and Vertical Speed	1328	0.04	mm/sec	Muzzle Speed
6. Downward Halt	1326	[QED]		

Muzzle Movement: Mode(n) mm = +/-A(n)*SIN[(2PI)*f(n)*t]

time (microsec)	time (seconds)	Mode 1 (mm)	Mode 2 (mm)	Mode 3 (mm)	Mode 4 (mm)	Mode 5 (mm)	Mode 6 (mm)	Mode 7 (mm)	Total Time (microsec)	SUM
0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.00000000	0.00000000	730	0.000000000
1	0.000001	0.000003	0.000023	0.000030	0.000001	0.000000	0.00000000	0.00000000	731	0.000009038
2	0.000002	0.000006	0.000047	0.000060	0.000001	0.000000	0.00000000	0.00000000	732	0.000018074
3	0.000003	0.000009	0.000070	0.000090	0.000002	0.000000	0.00000000	0.00000000	733	0.000027107
4	0.000004	0.000012	0.000093	0.000121	0.000003	0.000000	0.00000000	0.00000000	734	0.000036136
5	0.000005	0.000015	0.000117	0.000151	0.000003	0.000000	0.00000000	0.00000000	735	0.000045159
6	0.000006	0.000017	0.000140	0.000181	0.000004	0.000000	0.00000000	0.00000000	736	0.000054175
7	0.000007	0.000020	0.000163	0.000211	0.000005	0.000000	0.00000000	0.00000000	737	0.000063182
8	0.000008	0.000023	0.000187	0.000241	0.000005	0.000000	0.00000000	0.00000000	738	0.000072179
9	0.000009	0.000026	0.000210	0.000271	0.000006	0.000000	0.00000000	0.00000000	739	0.000081165
10	0.000010	0.000029	0.000234	0.000301	0.000007	0.000000	0.00000000	0.00000000	740	0.000090136
11	0.000011	0.000032	0.000257	0.000331	0.000007	0.000000	0.00000000	0.00000000	741	0.000099096
12	0.000012	0.000035	0.000280	0.000361	0.000008	0.000000	0.00000000	0.00000000	742	0.000108039
13	0.000013	0.000038	0.000304	0.000391	0.000009	0.000000	0.00000000	0.00000000	743	0.000116964
14	0.000014	0.000041	0.000327	0.000421	0.000009	0.000000	0.00000000	0.00000000	744	0.000125671
15	0.000015	0.000044	0.000350	0.000451	0.000010	0.000000	0.00000000	0.00000000	745	0.000134758
16	0.000016	0.000046	0.000374	0.000481	0.000011	0.000000	0.00000000	0.00000000	746	0.000143623
17	0.000017	0.000049	0.000397	0.000511	0.000011	0.000000	0.00000000	0.00000000	747	0.000152465
18	0.000018	0.000052	0.000420	0.000541	0.000012	0.000000	0.00000000	0.00000000	748	0.000161282
19	0.000019	0.000055	0.000444	0.000571	0.000013	0.000000	0.00000000	0.00000000	749	0.000170074
20	0.000020	0.000058	0.000467	0.000601	0.000013	0.000000	0.00000000	0.00000000	750	0.000178839
21	0.000021	0.000061	0.000490	0.000631	0.000014	0.000000	0.00000000	0.00000000	751	0.000187575
22	0.000022	0.000064	0.000514	0.000660	0.000015	0.000000	0.00000000	0.00000000	752	0.000196281
23	0.000023	0.000067	0.000537	0.000690	0.000015	0.000000	0.00000000	0.00000000	753	0.000204955
24	0.000024	0.000070	0.000560	0.000720	0.000016	0.000000	0.00000000	0.00000000	754	0.000213597
25	0.000025	0.000073	0.000583	0.000750	0.000016	0.000000	0.00000000	0.00000000	755	0.000222204
26	0.000026	0.000075	0.000607	0.000779	0.000017	0.000000	0.00000000	0.00000000	756	0.000230775
27	0.000027	0.000078	0.000630	0.000809	0.000018	0.000000	0.00000000	0.00000000	757	0.000239309
28	0.000028	0.000081	0.000653	0.000838	0.000018	0.000000	0.00000000	0.00000000	758	0.000247804
29	0.000029	0.000084	0.000677	0.000868	0.000019	0.000000	0.00000000	0.00000000	759	0.000256260
30	0.000030	0.000087	0.000700	0.000897	0.000020	0.000000	0.00000000	0.00000000	760	0.000264674
31	0.000031	0.000090	0.000723	0.000926	0.000020	0.000000	0.00000000	0.00000000	761	0.000273045
32	0.000032	0.000093	0.000746	0.000956	0.000021	0.000000	0.00000000	0.00000000	762	0.000281372
33	0.000033	0.000096	0.000770	0.000985	0.000021	0.000000	0.00000000	0.00000000	763	0.000289653
34	0.000034	0.000099	0.000793	0.001014	0.000022	0.000000	0.00000000	0.00000000	764	0.000297887
35	0.000035	0.000102	0.000816	0.001043	0.000023	0.000000	0.00000000	0.00000000	765	0.000306073
36	0.000036	0.000105	0.000839	0.001072	0.000023	0.000000	0.00000000	0.00000000	766	0.000314209
37	0.000037	0.000107	0.000863	0.001101	0.000024	0.000000	0.00000000	0.00000000	767	0.000322293
38	0.000038	0.000110	0.000886	0.001130	0.000024	0.000000	0.00000000	0.00000000	768	0.000330325
39	0.000039	0.000113	0.000909	0.001159	0.000025	0.000000	0.00000000	0.00000000	769	0.000338303
40	0.000040	0.000116	0.000932	0.001188	0.000025	0.000000	0.00000000	0.00000000	770	0.000346226
41	0.000041	0.000119	0.000956	0.001217	0.000026	0.000000	0.00000000	0.00000000	771	0.000354091
42	0.000042	0.000122	0.000979	0.001245	0.000026	0.000000	0.00000000	0.00000000	772	0.000361899
43	0.000043	0.000125	0.001002	0.001274	0.000027	0.000000	0.00000000	0.00000000	773	0.000369647
44	0.000044	0.000128	0.001025	0.001302	0.000028	0.000000	0.00000000	0.00000000	774	0.000377334
45	0.000045	0.000131	0.001048	0.001331	0.000028	0.000000	0.00000000	0.00000000	775	0.000384958
46	0.000046	0.000134	0.001072	0.001359	0.000029	0.000000	0.00000000	0.00000000	776	0.000392519
47	0.000047	0.000136	0.001095	0.001387	0.000029	0.000000	0.00000000	0.00000000	777	0.000400015
48	0.000048	0.000139	0.001118	0.001416	0.000030	0.000000	0.00000000	0.00000000	778	0.000407445
49	0.000049	0.000142	0.001141	0.001444	0.000030	0.000000	0.00000000	0.00000000	779	0.000414807
50	0.000050	0.000145	0.001164	0.001472	0.000031	0.000000	0.00000000	0.00000000	780	0.000422099
51	0.000051	0.000148	0.001187	0.001500	0.000031	0.000000	0.00000000	0.00000000	781	0.000429321
52	0.000052	0.000151	0.001210	0.001528	0.000032	0.000000	0.00000000	0.00000000	782	0.000436471
53	0.000053	0.000154	0.001234	0.001555	0.000032	0.000000	0.00000000	0.00000000	783	0.000443548
54	0.000054	0.000157	0.001257	0.001583	0.000033	0.000000	0.00000000	0.00000000	784	0.000450551
55	0.000055	0.000160	0.001280	0.001611	0.000033	0.000000	0.00000000	0.00000000	785	0.000457477
56	0.000056	0.000163	0.001303	0.001638	0.000033	0.000000	0.00000000	0.00000000	786	0.000464327
57	0.000057	0.000165	0.001326	0.001665	0.000034	0.000000	0.00000000	0.00000000	787	0.000471098
58	0.000058	0.000168	0.001349	0.001693	0.000034	0.000000	0.00000000	0.00000000	788	0.000477789
59	0.000059	0.000171	0.001372	0.001720	0.000035	0.000000	0.00000000	0.00000000	789	0.000484398
60	0.000060	0.000174	0.001395	0.001747	0.000035	0.000000	0.00000000	0.00000000	790	0.000490925
61	0.000061	0.000177	0.001418	0.001774	0.000036	0.000000	0.00000000	0.00000000	791	0.000497369
62	0.000062	0.000180	0.001441	0.001801	0.000036	0.000000	0.00000000	0.00000000	792	0.000503727
63	0.000063	0.000183	0.001464	0.001828	0.000036	0.000000	0.00000000	0.00000000	793	0.000509999
64	0.000064	0.000186	0.001487	0.001854	0.000037	0.000000	0.00000000	0.00000000	794	0.000516183
65	0.000065	0.000189	0.001510	0.001881	0.000037	0.000000	0.00000000	0.00000000	795	0.000522278
66	0.000066	0.000192	0.001533	0.001907	0.000038	0.000000	0.00000000	0.00000000	796	0.000528282
67	0.000067	0.000195	0.001556	0.001934	0.000038	0.000000	0.00000000	0.00000000	797	0.000534195
68	0.000068	0.000197	0.001579	0.001960	0.000038	0.000000	0.00000000	0.00000000	798	0.000540015
69	0.000069	0.000200	0.001602	0.001986	0.000039	0.000000	0.00000000	0.00000000	799	0.000545740
70	0.000070	0.000203	0.001625	0.002012	0.000039	0.000000	0.00000000	0.00000000	800	0.000551370
71	0.000071	0.000206	0.001648	0.002038	0.000039	0.000000	0.00000000	0.00000000	801	0.000556904