Tuning the Rifle Barrel and Load Together

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1. Why Tune for Bullet Launch with Zero Yaw?

If the rifle bullet is given a non-zero aeroballistic yaw and yaw-rate as it clears the muzzle, those attitude errors can only be amplified in their magnitudes by passage of that bullet through the muzzle-blast zone prior to commencing its ballistic flight. These amplified attitude errors will then be quite variable from one shot to the next. Even when firing through a wind-free atmosphere, any non-zero initial aeroballistic yaw results in an aerodynamic jump deflection of the entire remaining ballistic trajectory and in increased yaw-drag during the critical highest-drag beginning portion of the bullet's ballistic flight. This higher initial air-drag then also causes increased sensitivity to any ambient crosswinds immediately in front of the firing point.

Oddly enough, another effect of non-zero initial bullet yaw which has been observed by riflemen is a different response to crosswinds of opposite directions depending upon the sense of the rifling twist, right-hand or left-hand twist. The Coning Theory of Bullet Motions explains these different ballistic responses quite satisfactorily. Non-zero initial aeroballistic yaw-rate is an even worse offender than initial yaw itself in each of these negative aspects.

Because the CG of the typical rifle bullet is well forward of its point of last contact with the muzzle during its barrel exiting process, the bullet can exit without acquiring any non-zero yaw and yaw-rate only when the muzzle is stationary in inertial space in all cross-axis directions during bullet exit. If the muzzle of the rifle barrel is in cross-axis motion during bullet exit, it will tip the nose of the fired bullet in the direction opposite to that of the muzzle motion and also impart a yaw-rate which works to increase that non-zero attitude of the bullet's spin-axis orientation relative to its forward velocity direction. Subsequent passage of the bullet through the muzzle-blast zone amplifies both of these mechanically imparted yaw and yaw-rate attitude errors by significant and variable amounts.

The result of these initial attitude errors in ballistic flight are random spreads in target impact points (enlarged group sizes) and random variations in the reductions of their effective Ballistic Coefficient (BC) for the bullets being fired.

2. Recoil Effects on the Rifle Barrel

Initial-stage rifle recoil occurs only in reaction to the forward acceleration of the rifle bullet (and a very small portion of the powder charge) during the firing of the rifle. Rifle recoil does not commence until after the rifle bullet has been engraved with the rifling pattern and is free to accelerate down the bore. The line-of-action of this rearward recoil force acting on the rifle is coaxial with the bore of the rifle barrel. This recoil force acting on the rifle can be quantified at any time during the firing process after bullet engravement as the base-pressure driving the bullet forward multiplied by the cross-sectional area of the bore being obturated by that bullet. After initial engraving of the rifle bullet by the rifling lands, the forward-acting force of barrel friction is less than 2-percent of the bullet's driving force and is considered negligible here. This rearward recoil force on the rifle can be thought of as being applied to the rifle at the center of its breech face as a portion of the thrust of the cartridge case head against the breech face plus whatever rearward frictional force might obtain between the case walls and the chamber walls.

The CG of most shoulder-fired rifles being fired upright is located several millimeters (dCG) vertically below the axis of its bore. Thus, the recoil force briefly creates a barrel-bending torque acting vertically upward at the front face of the receiver upon the rear of the (assumed) free-floating rifle barrel. Here, we are analytically treating this upward torque impulse as being of a Gaussian shape (Normal probability distribution function) in the time domain, with its peak occurring at the instant of peak base-pressure driving the bullet forward in the rifle barrel. This analytical treatment allows the reasonably precise formulation of the barrel's dynamic response at its muzzle to this torque impulse being applied at its receiver end. We are not concerned here with the muzzle of the barrel being dragged straight rearward during recoil. Good rifle design and firing technique should eliminate any disturbance of the rifle barrel in the horizontal plane during firing, leaving only vertical plane motions to be formulated here.

The recoil torque impulse imparts a Gaussian shaped transverse shear-wave in a vertical plane into the material of the rifle barrel at its junction with the receiver face. Closely inspecting the base-pressure curve plotted against time in a good interior ballistics application such as QuickLOAD©, we see that it appears distinctly Gaussian in its excitation profile, having a "standard deviation," **sigma(time)**, given by its rise time from 60.65-percent to 100-percent of peak base-pressure.

The driving recoil torque also somewhat resembles the first half of a sine wave having a frequency which we term the "peak excitation frequency." The barrel's excitation spectrum in the frequency domain is given by the Fourier transform of the time-domain base-pressure curve. Fortunately, we know this excitation spectrum must then also be another Gaussian function of frequency centered at the peak excitation frequency and having a spread function **sigma(freq)** inversely related to that of its Gaussian time-domain driving function **sigma(time)**:

$$\mathbf{Sigma(freq) = 1/[(\pi^2)*Sigma(time)]}$$

If we model the rifle barrel mechanically as a "long, slender rod" of an isotropic steel material and having a uniform cross-section, we can use engineering handbook data to calculate its response to this forced shear-wave initial distortion. Specifically, the rifle barrel is modelled as a uniform, thick-walled, hollow cylindrical cantilever beam having a clamped end within its receiver junction and a free end at its muzzle.

An interior ballistics program gives us the time of bullet engravement (chamber pressure reaching 6,000 psi), the time of 60-percent of peak base-pressure (one sigma before the time of peak base-pressure), the time of peak base-pressure, and the time of bullet exit from the muzzle of our rifle barrel. We can calculate rather precisely the "signaling delay" between the time the shear-wave is introduced at the receiver/barrel junction and the earliest time when the muzzle of the barrel begins to react vibrationally to this recoil-driven torque impulse:

Signaling Delay = External Barrel Length/Shear-Wave Propagation Speed

Here we need the input Barrel Length (**L"**) as measured externally from receiver face to muzzle. We calculate the shear-wave propagation rate along the barrel as an un-tensioned "long, slender rod" from the properties of the steel barrel material. This speed is about **3054 meters per second** for 416R stainless steel rifle barrels.

The muzzle begins to vibrate sinusoidally in a transverse vertical plane as soon as the leading edge of the upward bending torque (shear-wave) signal reaches it. These vibrations are of multiple modes, all starting simultaneously, with each mode continuing at its own specific mode frequency which we calculate from handbook data for the barrel as a cantilever beam. Each of these mode frequencies is a naturally resonant (constructively reinforcing) frequency for transverse vibrations reflecting back and forth along the rifle barrel at about **3054 meters per second**. After peak excitation at the muzzle, these muzzle vibrations damp exponentially with a ring-damping time constant approximated by their round trip time up and down the length of the barrel. The actual muzzle motion is the ***algebraic sum*** of the sinusoidal vibration modes at their respective excitation amplitudes evaluated at the muzzle. The excitation amplitudes of the different vibration modes each vary as half Gaussian and half exponential decay functions of time which thus can be "factored out" analytically as a combined Pulse Width modulation function.

We seek to tune the muzzle exit times of our fired bullets to match a reversal time (halt) in vertical muzzle motion when the muzzle is momentarily stationary relative to the earth as a quasi-inertial reference. Known side benefits of this type of tuning are minimizing vertical (transverse) kick-velocities for our fired bullets and regularizing the muzzle pointing angles for each bullet launch.

We can achieve partial "compensation" for long-range gravity-drop variations due to variations in bullet launch velocities by tuning our mean bullet exit times to occur just earlier than an upward-to-downward muzzle halt, or just after a downward-to-upward muzzle reversal. This partial gravity-drop compensation is based upon an assumed, but quite likely, inverse correlation between variations in bullet launch velocities and variations in muzzle exit times for groups of shots fired sequentially.

3. Barrel Transverse Vibration Modes

For a uniform cantilever beam, its natural transverse shear-wave vibration **Mode(n)** frequencies are determined primarily by its beam length **L** and secondarily by its flexural rigidity **E*I** and its mass per unit length **A*ρ**, where **L** is the barrel's "vibrational length," **E** is Young's Modulus of Elasticity for the barrel steel, **I i**s the second moment of cross-sectional area (**A**) of the rifle barrel, and **ρ** is the density (mass per unit volume) of the barrel steel.

For a circularly cylindrical rifle barrel of uniform outside diameter **D** and caliber **d**, the second moment of area **I** is given by

$$\mathbf{I = (\pi/32)*(D^4 - d^4)}$$

and its uniform cylindrical cross-sectional area **A** is given by

$$\mathbf{A = (\pi/4)*(D^2 - d^2)}$$

The attached **Sheet 1** shows an image of a ***Data Input*** spreadsheet complete with units conversion utilities to allow inputs in British Engineering units to be converted conveniently into metric SI (MKS) units for these calculations.

The transverse vibration modes are numbered according to the count (**n**) of the vibration nodes (locations of zero vibrational amplitude) occurring over the beam length L for that vibrational mode shape. The (clamped) barrel/receiver joint is always considered to be a vibration node, while the (free) muzzle end is always a vibrational anti-node. Each segment of rifle barrel material vibrates independently and simultaneously at each mode frequency in a combined sinusoidal motion with mode amplitudes constrained within the individual mode-shape envelopes. In particular, the muzzle-end segment of the barrel vibrates transversely in a vertical plane according to the instantaneous ***sum*** of all of the first seven, or so, sinusoidal vibration modes, so that the muzzle vertical position **y(t)** is given at any time **t** after initial excitation time $\mathbf{t_0}$ by:

$$y(t) = PW(t) * \Sigma_{(n=1,\ 7)}\{A(n) * SIN[2*\pi*f(n)*(t - t_0)]\}$$

The initial muzzle excitation time t_0 is calculated as the sum of the input time of bullet engravement from an interior ballistics program and a calculated signaling delay based upon barrel length. All modes of muzzle vibrations initiate simultaneously at this time t_0.

The mode natural frequencies **f(n)** are given in hertz by:

$$f(n) = [1/(2*\pi)]*[a(n)/L^2]*SQRT[E*I/(A*\rho)]$$

The mode **n** frequency constants **a(n)** are taken from the handbook by Blevins, *Formulas for Natural Frequencies and Mode Shapes*, 1979, for a cantilever beam with clamped and free ends. They were measured experimentally.

The mode frequency calculations are shown for a worked example in the attached **Sheet 2**. The mode vibration Peak Amplitudes **A(n)** are found by multiplying a calculated **Mode 1** excitation amplitude by the relative amplitudes from the excitation spectrum for each mode frequency **f(n)**. The vibration equations for all modes (including **Mode 1**) are normalized to unit response at the muzzle. The overall excitation amplitude is calculated for **Mode 1** using basic physics, and the peak Relative Excitation Amplitudes **REA(n)** are found from the Gaussian function:

$$REA(n) = EXP\{-0.5*[(f(n) - f(peak))/sigma(freq)]^2\}$$

The mode shapes are also shown graphically in **Sheet 2** of the worked example. Note that with an upward driving torque applied at the receiver end of the barrel, the muzzle end is initially moved *upward* by each odd-numbered vibration mode and *downward* by each even-numbered vibration mode.

A Pulse Width modulation function of time, **PW(t)**, is also calculated based on the assumed Gaussian shape of the recoil function in the time domain and the time of peak muzzle disturbance t_{PM} calculated from data inputs.

$$PW(t) = EXP\{-0.5*[(t - t_{PM})/sigma(time)]^2\} \qquad [\text{for } t_0<t<t_{PM}]$$

The time of peak muzzle disturbance t_{PM} is calculated by summing the input time of peak base-pressure and the calculated signaling delay.

After the time of peak muzzle excitation, thedecreasing second half of this Gaussian pulse width function **PW(t)** is replaced with an exponential ring-decay function:

$$\mathbf{PW(t) = EXP[-(t - t_{PM})/(time\ constant)] \qquad [for\ t_{PM}<t]}$$

Muzzle vertical position in millimeters, **y(t),** is then calculated and plotted for each microsecond of possible bullet exit times.

These analytic functions of time are then analytically differentiated so that muzzle speeds in millimeters per second, **dy/dt**, can then also be calculated and plotted for each microsecond.

4. Barrel Taper and Muzzle Attachments

The calculations of muzzle position as a function of ongoing time **y(t)** are quite accurate for rifle barrels of constant outside diameter **D** and having no muzzle attachment. We have formulated simple extensions of these calculations to handle those target rifles having straight-tapered barrels and/or lightweight muzzle attachments, producing approximate results which should be good enough to be in the ballpark. Ultimately, there is no substitute for test firing in ambient shooting conditions for rifle tuning.

As shown in **Sheet 1**, we accept input of barrel mass, if that is accurately known, and calculate an Average Diameter to use for tapered barrels. Alternatively, we can accept a Midpoint barrel OD measurement and estimate barrel mass from that value. We also accept an unthreaded muzzle OD measurement and calculate a Vibrationally Effective Diameter to use in vibration calculations as its geometric mean with the calculated Average Diameter, which biases toward the smaller muzzle OD.

Without requiring more details being entered, we simply accept the mass of any muzzle attachment and calculate an adjusted Vibrational Length of the rifle barrel by extending its rod-measured internal length at its Average Diameter to match the total mass of the barrel and attachment. This approach seems to work well enough for muzzle brakes weighing a few ounces, but is speculative for heavier suppressors or barrel tuners. The vibrational coupling with a long, heavy suppressor attached to the muzzle via a single threaded joint is questionable in any case.

Eventual extensions of these calculation could perhaps include detailed calculations for various barrel profiles and muzzle attachments including barrel tuners and suppressors for those users willing to input more detailed measurement data.

It should be noted that similar calculations of barrel ID expansion pulses, as yet another type of barrel vibration mode, could be undertaken by those concerned with “Optimum Barrel Time” tuning. Those expansion pulses are also shear-wave vibrations propagating along the rifle barrel at the same rate as these vertical-plane transverse vibrations. However, those expansions initiate within the barrel at the location of peak base-pressure behind the bullet instead of at the receiver face, so they will remain several microseconds out of phase with these transverse muzzle vibrations.

5. Worked Example

The attached images show screenshots of a worked example using a “live” Excel workbook of four spreadsheets which is freely available upon request from the author as an email attachment. In addition to having access to a current version of Microsoft Excel, the user will also need access to a good interior ballistics program such as QuickLOAD©, which we routinely use in load development here. In particular, this analysis uses the QL definition of start time (**t = 0**) based upon the initiation of chamber pressure rising. Other interior ballistics programs might use time-since-sear-break, or time-since primer ignition, either of which would occur much earlier, and their event timings would require adjustment for use here. [Just note the time of chamber pressure rise and subtract it from all other event times.]

The example shown is for a known accurate load in a heavy-barrel 6.5x47 Lapua rifle using a 27-inch barrel of constant 1.250-inch OD and having no muzzle attachment. The bullet is a 153-grain Hornady A-Tip propelled by 39.2-grains of the new Alliant ReLoader-16 temperature compensated powder incorporating an anti-copper-fouling additive.

Sheet 1 shows the Data Inputs, **Sheet 2** shows the calculation of the Mode Frequencies and Shapes, **Sheet 3** shows the relative Excitation Spectrum, and **Sheet 4** shows the Calculated Results, including a graph of muzzle position **y(t)** in fractions of a millimeter versus time **t** in microseconds. The table of ***Event Times*** shows the QL-calculated bullet exit occurring at **75 microseconds** before the muzzle reverses its second upward movement and is starting back upward again. This timing match is excellent for launching these match-type bullets with near zero initial yaw and yaw-rate while also producing “compensated” long-range accuracy with this bullet and powder load. The calculated vertical muzzle speed is upward (positive) at **7.1 mm/sec**. Vertical muzzle speeds of less than **10 mm/sec** can be considered to be OK.

However, one not peremptorily dismiss these tiny calculated vibrational amplitudes and muzzle speeds as being so small that they can safely be ignored. They are perfectly capable of wrecking the expected performance of good rifles, bullets, and ammunition.

You will usually find that the peak excitation frequency (here **608.27 hz**) falls between the natural **Mode 2** frequency (here **434.58 hz**) and **Mode 3** frequency (here **1216.82 hz**). In this example, **Mode 2** is the dominant muzzle vibration with the faster **Mode 3** modulation providing the critical reversals in vertical direction muzzle motions.

By calculating a frequency discriminator equal to half the bullet’s muzzle velocity per unit barrel length and comparing that to the geometric mean of the **Mode 2** and **Mode 3** natural frequencies of the barrel, an early determination can be made as to which muzzle vibration mode will be dominant. **Mode 2** domination implies the recoil drive function mostly determines muzzle vertical motions, as typically occurs with short rifle barrels. **Mode 3** domination gets you into the region where resonant vibrations control muzzle motions, as often happens with long rifle barrels.

“Your results may vary,” as is often disclaimed.

6. Summary

If you find that you lack sufficient control authority in tuning your bullet weight and propellant load choices for bullet exit at or very near one of the plotted reversal times, your choice of rifle barrel length and weight simply cannot optimally fire bullets of your selected caliber, chambering, and bullet-weight range. Shortening an existing long barrel can usually allow proper load tuning with the desired bullet and powder. Using this analytical tool during rifle design can avoid making that costly mistake.

If you have never fired a properly tuned load in one of your rifles, you are in for an eye-opening treat. When each bullet is fired with essentially zero initial yaw, air-drag and crosswind sensitivity are greatly reduced and are more consistent, and target accuracy is greatly increased. “Benchrest accuracy” truly can be achieved by all riflemen. Benchrest competitors using 22-inch heavy varmint 6mm PPC barrels have been carefully tuning their loads to their rifle barrels to launch their bullets “straight” in prevailing match conditions for decades, since long before today’s interior ballistics programs were even available.