Rifling Twist-Rate Effect on Chamber Pressure

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Introduction

Interior ballistics calculations usually assume a rifle bullet will be fired through a barrel rifled at a standard twist-rate of about 40 calibers per turn, for example, a 30-caliber barrel rifled with a 12-inch twist. For some new applications we need to use barrels rifled at much faster twist-rates of about 20 calibers per turn where the spin-up inertia of the accelerating bullet might significantly increase the linear inertial resistance provided by the mass of the projectile. Riflemen know that inertial resistance is increased when selecting faster twist-rate barrels, but we have had no way to quantify the effects of that increase. We shall formulate this increased inertial resistance in terms of an increase in effective bullet mass (or weight) so that we can then use conventional interior ballistics programs like QuickLOAD© to calculate realistic chamber pressure curves with these fast-twist barrels. Using QuickLOAD for a representative cartridge and load, we find that both chamber pressure increases and the accompanying muzzle velocity losses are negligibly small until rifling twist-rates approach 10 calibers per turn.

Analysis

We will ignore bullet-to-bore friction in this formulation because it is difficult to quantify and because, whatever its effects, this friction is not primarily affected by our choice of rifling twist-rate in that barrel. We shall start our analysis just after the bullet has been engraved by the rifling lands in the throat of the barrel because very little rotation of the bullet is occurring during that brief high-stress mechanical process. Peak rotational acceleration of the bullet occurs along with peak linear acceleration at the time of peak base-pressure driving the bullet down the bore. The bullet has typically moved a few inches into the bore when this peak base-pressure occurs.

The instantaneous force **$F(t)$** driving the rifling engraved bullet down the barrel is given by the product of the instantaneous base-pressure **$P_b(t)$** and the cross-sectional area **S** of the bore:

$$F(t) = P_b(t)*S = [P(t) - 0.5*\rho*V^2]*S$$

where the second term in the brackets **$-0.5*\rho*V^2$** is a small dynamic pressure adjustment for bullet velocity **$V(t)$** applied to the instantaneous chamber-pressure **$P(t)$**. Because of this small dynamic effect, peak chamber pressure actually occurs slightly after peak base-pressure occurs.

According to Newton's Second Law of Motion and neglecting friction, most of this driving force **$F(t)$** is matched by the inertial force **$F_L(t)$** resisting the linear acceleration **d^2x/dt^2** of the bullet down the bore. This linear inertial force **F_L** is given in pounds by the product of the bullet's constant mass **m** and its linear acceleration **d^2x/dt^2**:

$$F_L = m*d^2x/dt^2$$

where, in British Engineering Units,

$$m = (\text{Bullet Weight in pounds})/g$$

and $$g = 32.174\ \text{feet/second}^2.$$

The position **x** (in feet) of the bullet along the axis of the bore defines our **x-axis** here.

Still neglecting friction, the remainder of the driving force **$F - F_L$** is matched by an instantaneous axial force **F_A** spinning-up the bullet due to the rifling twist. This axial force **F_A** is attributable to the second moment of angular inertia **I_x** of the bullet's mass distribution about this same **x-axis** and the second time derivative **$d^2\Theta/dt^2$** of the bullet's angular orientation **Θ**.

If the caliber of the bore is **d**, we can define the ***twist-rate*** of the rifling as **$n*d$** where **n** is the number of ***calibers per turn*** of that rifling. Each full turn of the rifling pattern represents **$2*\pi$** radians of angular motion in **Θ**. The engraved bullet remains mechanically locked into the rifling as it moves, so that, at any time **t** while the bullet is traversing the rifle barrel:

$$\Theta(t) = [2*\pi/(n*d)]*x(t).$$

Since the twist-rate **n** and caliber **d** are taken as constants here, after twice differentiating with respect to time, we have:

$$d^2\Theta/dt^2 = (2\pi/nd)*d^2x/dt^2.$$

The second moment of inertia I_x of the mass distribution of the bullet about its **x-axis** can be expressed as:

$$I_x = m*k^2$$

where **m = Mass** of the bullet

and **k = Radius of Gyration** of the mass distribution of the bullet about its **x-axis.**

From the rotational form of Newton's Second Law of Motion, the inertial resistance to spinning-up the bullet provides a reactive torque **T** about the **x-**axis such that:

$$T = I_x*d^2\Theta/dt^2$$

or, after substituting,

$$T = [m*k^2]*[(2\pi/nd)*d^2x/dt^2].$$

Now, we need to relate this rotational inertial resistance torque **T** about the **x-axis** of the bore to its equivalent linear resistance force F_A acting back along that axis, still neglecting friction but taking into account the assumed constant helix angle **α** due to the twist of the rifling. By "unrolling" one full turn of the rifling twist, we see that:

$$\mathrm{Tan}[\alpha] = (\pi*d)/(n*d) = \pi/n.$$

Thus **α** equals **4.491 degrees** when **n** is **40 calibers per turn**, and increases to **8.927 degrees** at **n = 20**.

Fortunately, we recognize this twisting linear motion mechanism as being equivalent to the relationship between the tightening torque **T** applied to a threaded fastener and its resultant axial clamping force F_A in the absence of friction. From textbooks on mechanics, this relationship is given as:

$$F_A = (2\pi/nd)*T$$

where here our twist-rate **n*d** takes the place of the ***thread pitch*** **p** of the threaded fastener. This formulation merely expresses the equivalence of the instantaneous rates of linear and rotational kinetic energy changes occurring in the absence of friction losses:

$$nd*F_A(t) = 2\pi*T(t).$$

Substituting our expression for **T** above and rearranging, we have:

$$F_A = (2\pi/n)^2 *(k/d)^2 *m*d^2x/dt^2.$$

Summing the two reaction forces, we now have the needed formulation:

$$F = F_L + F_A$$

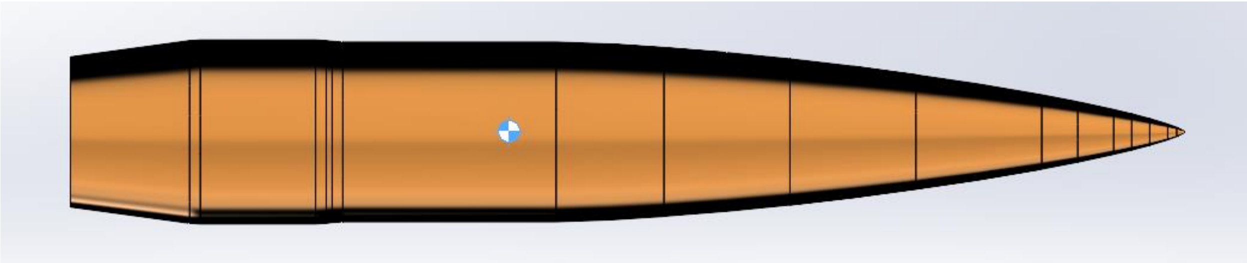
or

$$F = [1 +(2\pi/n)^2 *(k/d)^2] * m*d^2x/dt^2.$$

From the mass properties accurately calculated in a CAD program for the dual-diameter, solid monolithic copper Ultra-Low-Drag bullets of my own design, their radius of gyration **k** values about their spin axes, expressed in groove-diameter calibers d_G, is

$$k/d_G = 0.329868.$$

This dual-diameter copper ULD bullet design is shown below. The **0.45-caliber** drilled base is not shown in this view. This bullet design is missing a shell of copper over the front 80-percent of its length compared to a conventionally designed bullet. The thickness of the missing copper shell is equal to the nominal groove depth of the rifling pattern, or about **0.004 inch** for most small caliber rifle bullets. Because of this, a more conventional bullet design would have somewhat larger second moment I_X and radius of gyration **k**.



For a homogeneous solid cylindrical slug projectile, mechanical reference handbooks give a radius of gyration about its spin-axis k_X as **0.707107*r**. So, the largest possible radius of gyration for any homogeneous solid bullet expressed in groove-diameter calibers $d_G = 2*r$ is

$$\mathbf{k/d_G = 0.353553\ calibers}.$$

Thus, we can estimate the likely intermediate ratio of **k/d** to be approximately

$$\mathbf{k/d \approx 0.345\ calibers}$$

for a more conventionally designed, but aerodynamically similar, solid ULD or VLD rifle bullet without the base-drilling.

Results

Let us examine the second term in the square brackets of our formulation above, the ***equivalent fractional mass increase of the bullet when taking spin-up inertia into account***. For a traditional rifling twist-rate of **40 calibers per turn**, we calculate:

$$\mathbf{(2\pi/40)^2 *(0.345)^2 = 0.0029368\ or\ 0.294\ percent}$$

Since about this amount of standard spin-up inertia must already be included in the ballistics data used in applicable interior ballistics programs such as QuickLOAD©, we should modify our formulation here to compensate for this fact:

$$\mathbf{F = [0.9970632 +(2\pi/n)^2 *(k/d)^2] * m*d^2x/dt^2}.$$

Now, when we need to calculate the equivalent bullet mass (or weight) to enter into QuickLOAD for calculating the pressure effects when using a fast twist barrel with **n = 20 calibers per turn**, for example, we simply multiply the bullet mass (or weight) by a factor of:

$$\mathbf{0.9970632 +(2\pi/20)^2 *(0.345)^2 = 1.008810}$$

Thus, for example, one could simply enter into QuickLOAD an effective bullet weight of **302.64 grains** when firing a **300 grain** bullet from a very fast-twist **20 calibers per turn** rifle barrel. All QuickLOAD calculated outputs would then be properly adjusted for the use of this very fast-twist barrel. In fact, at the expense of incorporating an additional required user input of *rifling twist-rate*, this effective bullet mass adjustment calculation could easily be added into QuickLOAD© itself.

The effects on the chamber pressure curve of increasing the effective bullet weight by **0.881 percent** here due to ***doubling*** the standard twist-rate will not be nearly as large as many had feared.

The table below shows effective bullet weights to be used for **200-grain 30-caliber** bullets when they are fired from barrels having different twist-rates. The resulting peak chamber pressure effects with at typical **60,000 PSI** maximum load are also shown. As can be seen, the typical increases in peak chamber pressure as calculated in QuickLOAD© are only about **80-percent** of those rather small calculated effective bullet weight increases. The small decreases in calculated muzzle velocity when reasonably faster twist-rates are selected are practically negligible until the rifling twist-rates selected approach an extreme of just **10 calibers per turn**.

Effective bullet weights for 200-gr 30-caliber bullets with different twist-rates							
	[kx = 0.345 calibers]		[d = 0.3000 inches]				
Rifling Twist (inches per turn)	15.00	12.00	10.00	8.00	6.00	4.50	3.00
Calibers per turn (n)	50.00	40.00	33.33	26.67	20.00	15.00	10.00
Helix Angle in Degrees	3.595	4.491	5.384	6.719	8.927	11.829	17.441
Bullet Mass (or Weight) Adjustment	0.998943	1.000000	1.001292	1.003671	1.008810	1.017947	1.044052
Effective Bullet Weight (grains)	199.79	200.00	200.26	200.73	201.76	203.59	208.81
Percent Difference in Bullet Weight	-0.106	0.000	0.129	0.367	0.881	1.795	4.405
Peak Chamber Pressures (QL, PSI)	59,950	60,000	60,060	60,180	60,440	60,900	62,180
Percent Difference in PSI	-0.083	0.000	0.100	0.300	0.733	1.500	3.633
Muzzle Velocity (FPS)	2796	2796	2795	2794	2792	2788	2776
Percent Difference in MV	0.0000	0.0000	-0.0358	-0.0715	-0.1431	-0.2861	-0.7153